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February 19 Lecture

Textbook Reference:
- *Vector Calculus*, Colley, 4th Edition: §1.5

Affine Geometry: lines & planes

Learning Objectives:
- *Understand how to determine a parametric description of a line.*
- *Understand how to determine the equation of a plane.*
- *Understand the difference between a parametric description and an equation description of a geometric object.*

Parametric descriptions of lines: given a point P and direction vector $\overrightarrow{u}$ we can give a parametric description of the line through P parallel to $\overrightarrow{u}$:

$$\underline{r}(t) = \overrightarrow{OP} + t\overrightarrow{u}, \quad t \in \mathbb{R}.$$

1. Let $P = (1, 1, 1)$, $Q = (2, 0, 1)$. Give the parametric description of the line L passing through P and Q.

2. Consider the line with parametric description

$$\underline{r}(t) = \begin{bmatrix} 5t \\ 2 - 5t \\ 1 \end{bmatrix}, \quad t \in \mathbb{R}$$

   Explain why this line is equal to L. (**Parametric descriptions are non-unique**)

3. Does the point $R = (2, -1, 2)$ lie on L? How can you check?

Equations of planes: Let P be a point, $\overrightarrow{n}$ a direction vector. There is a unique plane containing P and perpendicular to $\overrightarrow{n}$ (a **normal vector**). It is defined by the equation

$$\overrightarrow{n} \cdot \overrightarrow{x} = \overrightarrow{n} \cdot \overrightarrow{OP}, \qquad \text{where } \overrightarrow{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

1. Determine the equation of the plane containing the points $P = (1,0,0)$, $Q = (0,1,0)$, $R = (0,0,1)$.

2. What is the equation of the plane containing the point $(2,5,1)$ and perpendicular to the line L from the first set of problems above?

3. Determine whether the plane defined by the equation $2x + 5z = 10$ and the line $\overrightarrow{r}(t) = \begin{bmatrix} 1 \\ 2+t \\ 2t \end{bmatrix}$, $t \in \mathbb{R}$, are perpendicular, parallel, or neither. (*How can you quantify these possibilities using a normal vector?*)

Another distance formulae:

1. Let Π be the plane with normal vector $\overrightarrow{n}$ passing through P. Let Q be any point. Explain why the distance from Q to Π is

$$\frac{|\overrightarrow{PQ} \cdot \overrightarrow{n}|}{|\overrightarrow{n}|}$$

(*Hint:* it will be useful to draw a diagram representing the problem)

2. Let Π be the plane with normal $\overrightarrow{n} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ passing through $P = (1, 0, 0)$. Let $Q = (-1, 2, 2)$.

   (a) Explain why Q is not on the plane Π.

   (b) Determine the point of intersection between Π and the line through Q in the direction $\overrightarrow{n}$

Misc:

1. Let Π and Π' be non-parallel planes, and let $\overrightarrow{n}$, $\overrightarrow{n}'$ be corresponding normal vectors. In your groups, give a geometric explanation for why the line of intersection $\Pi \cap \Pi'$ is parallel to $\overrightarrow{n} \times \overrightarrow{n}'$. If possible, use the space below to provide a diagram supporting your explanation.