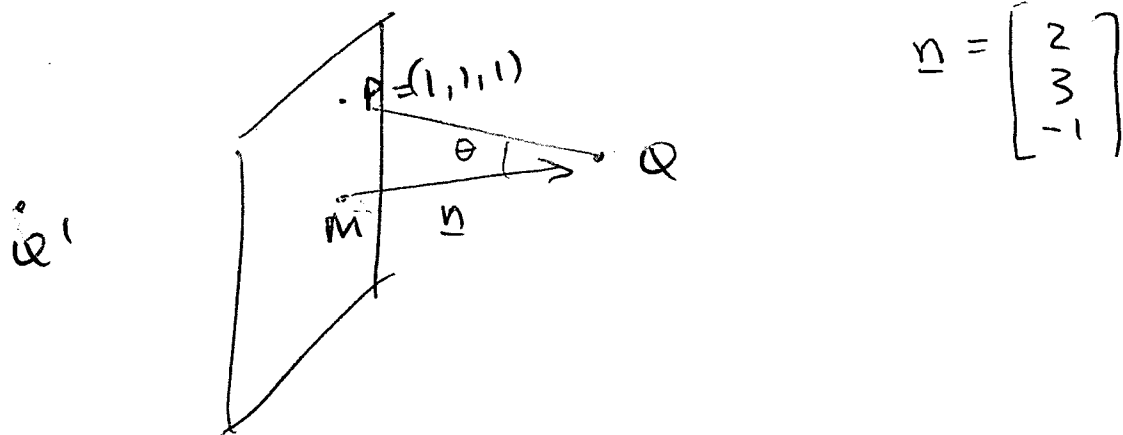


# PRACTICE EXAM I: MATH 223AB

3b)  $Q = (4, 5, -3)$ ,  $Q' = (a, b, c)$



• DISTANCE FROM  $Q$  TO  $\pi$ :  $\frac{|\vec{PQ} \cdot \vec{n}|}{|\underline{n}|}$

$$= \frac{22}{\sqrt{14}}$$

$$\Rightarrow |QQ'| = \frac{44}{\sqrt{14}}$$

## SOLUTION ①

\* Line through  $QQ'$  parameterised as

$$\underline{r}(t) = \begin{bmatrix} 4 + 2t \\ 5 + 3t \\ -3 - t \end{bmatrix}$$

Determine midpoint  $M$ : it's the point

$Q$  form  $\begin{bmatrix} 4 + 2t \\ 5 + 3t \\ -3 - t \end{bmatrix}$

substituting

$$2x + 3y - z = 4$$

$$\text{ie } 4 = 2(4+2t) + 3(5+3t) - (-3-t)$$

$$= 26 + 14t$$

$$\Rightarrow t = -\frac{22}{14} = -\frac{11}{7}$$

$$\Rightarrow M = \begin{bmatrix} 4 - 22/7 \\ 5 + 33/7 \\ -3 + 11/7 \end{bmatrix} = \begin{bmatrix} 6/7 \\ 2/7 \\ -10/7 \end{bmatrix}$$

$$\vec{OQ'} = \vec{OQ} + 2\vec{QM}$$

$$= \begin{bmatrix} 4 \\ 5 \\ -3 \end{bmatrix} + 2 \cdot \begin{bmatrix} 6/7 \\ 2/7 \\ -10/7 \end{bmatrix}$$

$$= \begin{bmatrix} -16/7 \\ -31/7 \\ 1/7 \end{bmatrix}$$

$$\begin{aligned} a &= -11/7 \\ b &= -31/7 \\ c &= 1/7 \end{aligned}$$

for some  $t$ ,

SOLUTION ②

$Q'$  is point of form  $\begin{bmatrix} 4+2t \\ 5+3t \\ -3-t \end{bmatrix}$  satisfying

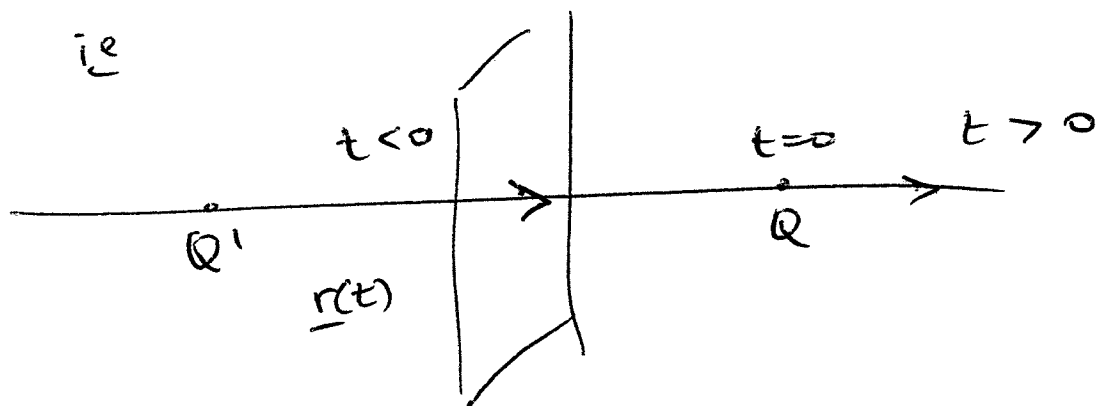
$$\frac{44}{\sqrt{14}} = |\vec{OQ'}| = \left| \begin{bmatrix} 2t \\ 3t \\ -t \end{bmatrix} \right|$$

$$\Rightarrow \frac{44^2}{14} = 4t^2 + 9t^2 + t^2 = 14t^2$$

$$\Rightarrow t^2 = \left(\frac{44}{14}\right)^2 \Rightarrow t = \pm \frac{44}{14} = \pm \frac{22}{7}$$

$$\text{Since } \vec{PQ} \cdot \vec{n} = \begin{bmatrix} 3 \\ 4 \\ -4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \\ = 22 > 0$$

$\vec{PQ}$  and  $\vec{n}$  point to same side  
of plane  $\pi$



$$\text{So, } t = -\frac{22}{7}$$

$$\Rightarrow Q' = \begin{bmatrix} 4 - 44/7 \\ 5 - 66/7 \\ -3 + 22/7 \end{bmatrix} \\ = \begin{bmatrix} -16/7 \\ -31/7 \\ 1/7 \end{bmatrix}$$