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April 4 Lecture

Textbook Reference:
- *Vector Calculus*, Colley, 4th Edition: §2.6

Directional Derivative

Learning Objectives:
- *Learn how to compute the directional derivative.*
- *Learn how to compute the tangent line/plane to a level curve/surface.*

Directional Derivatives: Given a differentiable function $f : X \subseteq \mathbb{R}^n \to \mathbb{R}$, $\underline{a} \in X$, $\underline{v} \in \mathbb{R}^n$, the **directional derivative of f at $\underline{a}$ in the direction $\underline{v}$** is

$$D_{\underline{v}} f(\underline{a}) = \nabla f(\underline{a}) \underline{v}$$

1. Compute the directional derivative of $f(x, y) = x^2 + 3xy + y^2$ at the point $(2, 1)$ in the direction that points towards the origin.

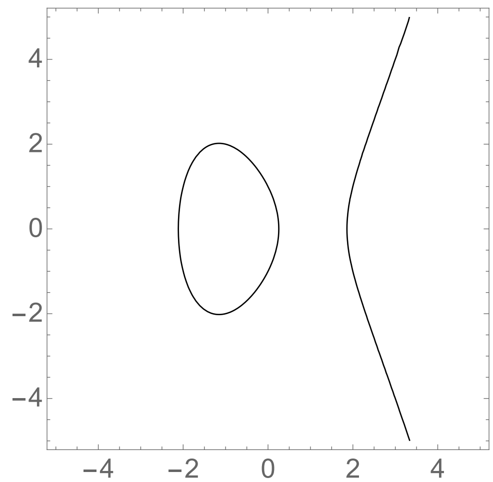
2. Find a nonzero vector $\underline{v} \in \mathbb{R}^2$ so that $D_{\underline{v}} f(2, 1) = 0$

3. Let $\underline{w} \in \mathbb{R}^2$ be a vector satisfying $\underline{w} \cdot \underline{v} > 0$ and $\underline{w} \cdot \begin{bmatrix} -2 \\ -1 \end{bmatrix} < 0$. Is f increasing in the direction $\underline{w}$ at $(2, 1)$? (*Recall that, if $\underline{a} \cdot \underline{b} > 0$ (resp. < 0) then angle between $\underline{a}$, $\underline{b}$ is acute (resp. obtuse)*)

Tangent lines/planes of level sets: Let S be a level set of $f : X \subset \mathbb{R}^n \to \mathbb{R}$, where $n = 2, 3$. This means $S = \{\underline{x} \mid f(\underline{x}) = c\}$, for some c. Then, $\nabla f(\underline{a})$ is

- perpendicular to the tangent line to S at $\underline{a}$ $(n = 2)$,
- normal to the tangent plane to S at $\underline{a}$ $(n = 3)$.

1. Consider the curve $y^2 = x^3 - 4x + 1$.



(a) Find a function $h(x, y)$ so that the curve is a level curve of h

(b) Compute the tangent line to the curve at $(2, 1)$.

(c) Determine the point (x, y), $y > 0$, where the tangent line is horizontal.

2. Consider the paraboloid $z = x^2 + y^2$. This is the graph of the function $f(x, y) = x^2 + y^2$.

   (a) Find a function $g(x, y, z)$ so that the paraboloid is a level set of g.

   (b) Determine the tangent plane to the paraboloid at $(1, 2, 5)$

   (c) Suppose that $z = f(x, y)$ is a surface. Generalise your approach above to determine the tangent plane to the surface at $(a, b, f(a, b))$.