Middlebury College

Multivariable Calculus: Spring 2018

Homework

Due April 4, 6pm

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Some thoughts and advice:

- You should expect to spend at least $1-2$ hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site). However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets together. **This will make your life easier!** However, you must write your solutions *on your own* and *in your own words*.

⋆ ⋆ ⋆ You **are not allowed** to use any additional resources (e.g. solution manuals, `stackoverflow` etc). If you are concerned then please ask.

- The problems in parentheses are for extra practice and optional (in particular, they do not need to be submitted). **Problems for submission from the textbook are underlined. Additional problems not from the textbook must be submitted**.

  To gain mastery of a topic you should expect to attempt a significant proportion of the problems in the textbook ($> 60\%$(!)).
- Answers to odd-numbered exercises are at the back of the textbook. However, you need to submit a worked solution and provide justification for how you determined the answer.

- **Read/recap:** §2.4, 2.5
- **Problems:** §$X.y$ refers to Chaper X, Section y of *Vector Calculus*, by Colley (4th Edition). All problems are taken from the 4th Edition.

  §2.4: (1), 2, (3), 5, (7), 9, (10), (11), 18, (19), 23, (24), (27)
  §2.5: (1), (2), 4, 7, (11), 13, 19, (20), 21, (22), (24), (25), 28, 30

Problem A: The level curve diagram from March 21 Homework is given on the following page. This is the level curve diagram of the function

$$f(x,y) = x(x-1)(x-2) + (y-1)(x-y)$$

Plotted on the level curve diagram is the point $A = (0,1)$.

1. Consider the path $\underline{r}(t) = (t,1)$, $t \in \mathbb{R}$.

   (a) Using the chain rule compute $\frac{d}{dt}(f \circ \underline{r})(0)$.

(b) Explain geometrically why your answer in (a) is > 0, $= 0$ or < 0.

(c) Describe the relationship between $\frac{d}{dt}(f \circ \underline{r})$ and $\frac{\partial f}{\partial x}$.

2. Now consider the path $\underline{s}(t) = (t, 2t + 1)$, $t \in \mathbb{R}$. Observe that $\underline{s}(0) = A$.

(a) Using the chain rule compute $(f \circ \underline{s})'(t)$.

(b) The graph of $f(x, y)$ models a mountain range in Vermont. You are walking along a path on this mountain range whose (local) coordinates are described by $(\underline{s}(t), (f \circ \underline{s})(t))$, where t denotes the time in hours before/after 12pm, April 1 2018. For example, at 9am, April 1 2018 you are at $(-3, -5, -72)$.

What is your highest/lowest elevation if you walk from 9am to 6pm on April 1 2018?

(c) Will you be tired at 12pm?

(d) What is the relationship between the graph of $(f \circ \underline{s})(t)$, $-3 \leq t \leq 6$, and your elevation during your walk?