Writing Assignment # 3

One of the major themes of this course has been extending concepts from single-variable calculus into multivariable calculus, and realizing that while many of the basic ideas are similar, some very strange things can occur in higher dimensions. Many of these strange things required us to build unusual and complicated functions with discontinuities or undefined limits. Here we will see an example of a situation where a particular idea from single-variable calculus can't generalize to higher dimensions, **even with simple polynomial functions**.

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1. Working in the world of Calculus 1: Suppose $f(x)$ is a polynomial function in one variable which has exactly one critical point: a local minimum. Explain why this local minimum must also be a **global** minimum for $f(x)$.

2. Consider now the function $f(x, y) = x^2(y+1)^3 + y^2$, a polynomial function in two variables. Plot this function on Wolfram Alpha in the window from -0.5 to 0.5, using the command
   `plot f(x, y) =` $\texttt{x}^2\texttt{(y+1)}^3$ `+` $\texttt{y}^2$ `from y=-0.5 to 0.5.`

   Looking at the contour plot, how many stationary points appear? What kind are they? Include the contour plot with your assignment, and discuss how you make this determination.

3. Using the calculation methods discussed in class, show that $(0, 0)$ is the only stationary point for $f(x, y)$, and that it is a local minimum.

4. Show that $(0, 0)$ is not a **global** minimum for $f(x, y)$, by proving that there are many points (a, b) with $f(a, b) < f(0, 0)$. How is this different from what you showed in Part (1)?

5. Explain as deeply as you can why your explanation for Part (1) doesn't hold up in higher dimensions. It may help to plot the function over a wider window to get a better sense of its true behavior. What is going on? Why doesn't the same argument work?