

SUMMARY OF SERIES TESTS

- (1) **Test for Divergence** *always check first*
- (2) **Geometric Series Test** (GST) *detect by looking at series*
- (3) **p -series Test** *detect by looking at series*
- (4) **Comparison Test** *terms must be nonnegative*
- (5) **Ratio Test** *terms must be positive*
- (6) **Integral Test** *terms must be nonnegative*
- (7) **Alternating Series Test** (AST) *only applies to alternating series*
- (8) **AC \implies C** *applies to any series*

Tips:

- (a) Always write out first few terms of series.
- (b) **Don't forget:** a series converges is equivalent to its sequence of partial sums converging.
- (c) If a series is alternating, try AC \implies C first; then try AST.
- (d) If a series has factorials (e.g. $(k+2)!$) or exponentials (e.g. 7^k) appearing in its (positive) terms, try Ratio Test. Then try Comparison Test.
- (e) In general, the Ratio Test will be inconclusive for series whose terms are rational functions of k e.g. $\sum_{k=1}^{\infty} \frac{2k^2}{k^3 + 2k + 5}$.
- (f) Generally, always try to compare with geometric series or p -series.
- (g) Try to simplify the formula for terms e.g. $\frac{k!}{(k+1)!} = \frac{1}{k+1}$.
- (h) If a series has some negative terms, some positive terms, but it's not alternating, your **only** choice is AC \implies C.
- (i) We have **not** seen a Squeeze Theorem for series.
- (j) Ensure you write down the tests you are using, and at what point.
- (k) When using Comparison Test, ensure you explain why the series you are comparing to is convergent/divergent.