Dear All,

As promised, I have written an itemized list of topics we've covered in Math 122 since Midterm 1. This list of topics (and the proportion of time we've spend on them) will align with the problems you will see on Wednesday's midterm exam. In studying for the exam, please note that I consider the homework exercises and the examples I've covered in lecture to be the best source of practice problems. If you know how to approach each problem, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution, you should perform well on the exam.

### Exam Format

(I) Unless otherwise arranged, you have two hours to complete the exam.

(II) Calculators are **not permitted** for use during the exam.

(III) The exam is **closed book**: notes, textbooks, computers, mobile devices, listening devices are **not permitted** for use during the exam.

(IV) There will be a total of five problems on the exam. To receive full credit you will need to provide correct and complete solutions to all five problems.

(V) There will be one problem consisting of several True/False subproblems.

(VI) There will be one short-answer problem consisting of several short, computational subproblems.

(VII) There will be three long-answer problems. Each long-answer problem may have multiple parts.

(VIII) A student with a solid grasp of the material and excellent problem-solving ability should be able to complete the exam in $< 1$ hour.

# Theory

## Definitions/Concepts/Results

You should know all definitions/concepts/results from lecture (and used in homework) precisely, including all of the theorems, propositions and corollaries we've covered. If I've given a proof of something in class, you should know it (or have a thorough understanding of the details). Here is a list of concepts/definitions/results we've seen since the previous midterm:

1. The Euclidean plane $\mathbb{R}^2$, Euclidean 3-space $\mathbb{R}^3$ and Euclidean $n$-space $\mathbb{R}^n$.
2. The Euclidean distance between points $P$ and $Q$ (in $\mathbb{R}^2$ or $\mathbb{R}^3$).
3. The equations of a sphere in $\mathbb{R}^3$ with center $(x_0, y_0, z_0)$ and radius $r > 0$.
4. The notion of a vector (as a quantity with magnitude and direction).
5. Vector addition/subtraction and scalar multiplication.
6. The component form of a vector in $\mathbb{R}^2$, $\mathbb{R}^3$.
7. You should know how to perform addition/subtraction/scalar multiplication of vectors in terms of components.
8. You should know the algebraic properties of vectors.
9. You should know the definition of the norm/length/magnitude $|\mathbf{a}|$.
10. The "standard" unit vectors $\mathbf{i}, \mathbf{j}$ in $\mathbb{R}^2$ and $\mathbf{i}, \mathbf{j}$, $\mathbf{k}$ in $\mathbb{R}^3$.

11. You should know the algebraic definition of the dot product.
12. You should know the algebraic properties of the dot product.
13. You should know the geometric formula for the dot product.
14. You should know the concept of orthogonality/perpendicularity of vectors and how orthogonality/perpendicularity is characterized by the dot product.
15. You should know the definition of the scalar and vector projections and properties thereof; in particular, see Homework 7, Problem 2.
16. The cross product and its geometric properties.
17. The algebraic formula for the cross product.
18. The parametric descriptions of lines in $\mathbb{R}^2$ and $\mathbb{R}^3$.
19. You should know the equation of a plane.
20. Know how to understand the shape of a simple surface using cross sections, contours, at the level of examples seen in class.
21. You should know what a function of two/three variables is and its domain and range.
22. You should know what the graph of $f(x, y)$ is.
23. You should know what the contour plot/level curve diagram of a function $f(x, y)$ is and its interpretation as a topographical description of the graph of $f(x, y)$.
24. You should know the concept of a two variable limit and its relationship to limits along parameterized curves cf. Propositions A, B, C in Homework 8.
25. You should know the definition of partial derivatives.
26. You should know the definition of the local linearization of a function $f$ at a point $(a, b)$.
27. The concept of a directional derivative.
28. The gradient and its relationship to the directional derivative.

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# Computation

In general, you should know how to work problems connected to each key concept discussed in the previous section. As I said before, if you understand all of the lecture material and all of the homework (both the "turn in" and "do not turn in" problems), are able to work quickly and efficiently, you should perform well on the exam. Here is an incomplete list of things we have done since Midterm 1:

1. Given a vector $\mathbf{v}$, you should know how to find a unit vector $\mathbf{u}$ with the same direction as $\mathbf{v}$.
2. You should know how to express any vector in $\mathbb{R}^2$ or $\mathbb{R}^3$ as linear combination of $\mathbf{i}, \mathbf{j}$ in $\mathbb{R}^2$ and $\mathbf{i}, \mathbf{j}$, $\mathbf{k}$ in $\mathbb{R}^3$. For example, if $\mathbf{a} = (a_1, a_2, a_3)$, then $\mathbf{a} = a_1(1, 0, 0) + a_2(0, 1, 0) + a_3(0, 0, 1) = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$.
3. You should know how to compute dot and cross products and norms/magnitudes.
4. Given two non-zero vectors $\mathbf{u}$ and $\mathbf{v}$, you should know how to use vector projections to express $\mathbf{u}$ as a sum of orthogonal vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ where $\mathbf{u_1}$ is parallel to $\mathbf{u_2}$. Both should be given in terms of $\mathbf{u}$ and $\mathbf{v}$. See Problem 2, Homework 7.

5. You should be able to compute point-to-plane distances. See, e.g., Problem 3 on Homework 6.
6. You should be able to describe the parameterization of a line through a point $P$ in a specified direction.
7. You should be able to describe a line through two distinct points $P$ and $Q$.
8. You should be able to give the equation of a plane passing through a point $P$ and normal/orthogonal to a vector $\mathbf{N}$.
9. You should be able to give the equation of a plane containing three distinct points $P$, $Q$ and $R$.
10. You should be able to determine the domains of basic functions of two variables.
11. You should be able to graph and produce contour plot/level curve diagrams of functions of two variables.
12. You should be able to compute partial derivatives.
13. You should be able to compute the local linearization/linear approximation of a function and use it to estimate the function.
14. You should be able to show that a limit does not exist by taking limits along distinct curves.
15. You should be able to interpret information about the partial derivatives of a function in terms of the meaning of such a function for a particular application (think about the room with the heater from HW 6, or the bone density problem from HW8).

**Review Problems** The following problems will provide you with good practice for the midterm. If you want more problems then please feel free to ask.

1. Chapter 12 Summary (p.697): 1, 3, 9, 11, 13, 19, 23, 25, 35, 37, 43, 51-55, 58, 59
2. Chapter 13 Summary (p.771): 1, 4-6, 11-13, 14, 19, 21-32, 34, 35, 37, 39, 44-46, 48-50, 54-57, 62, 64, 66, 68, 69a 73, 75, 79