This Midterm 1 Overview and Review serves three purposes:

- To provide an overview of the format of Midterm 1.
- To provide a detailed list of topics you are expected to know for Midterm 1 and can expect to be tested on.
- To provide some practice problems to aid your studying.

Exam Format

(I) Unless otherwise arranged, you have two hours to complete the exam.

(II) Calculators are **not permitted** for use during the exam.

(III) The exam is **closed book**: notes, textbooks, computers, mobile devices, listening devices are **not permitted** for use during the exam.

(IV) There will be a total of five problems on the exam. To receive full credit you will need to provide correct and complete solutions to all five problems.

(V) There will be one problem consisting of several True/False subproblems.

(VI) There will be one short-answer problem consisting of several short, computational subproblems.

(VII) There will be three long-answer problems. Each long-answer problem may have multiple parts.

(VIII) A student with a solid grasp of the material and excellent problem-solving ability should be able to complete the exam in < 1 hour.

Outline of Topics to Know

Theory

Except for the ϵ-N definition of the limit, you should know all definitions/concepts from lecture (and used in homework) precisely. In particular, you should know the definitions of the following terms/concepts and how they are used:

(a) The definition of a sequence of numbers $\{a_n\}$.

(b) What it means for a sequence $\{a_n\}$ to converge to a number a. Equivalently, what $\lim_{n\to\infty} a_n = a$ means.

(c) You should know what it means for a sequence to diverge (and to diverge to $\pm\infty$).

(d) What it means for a sequence to be bounded, bounded above, bounded below.

(e) What it means for a sequence to be non-increasing/non-decreasing/increasing/decreasing/monotonic.

(f) You should know what a series $\sum_n a_n$ is.

(g) Given a series $\sum_n a_n$, know what its sequence of partial sums $\{S_n\}$ is.

(h) You should know what it means for a series $\sum_n a_n$ to converge.

(i) You should know what it means for a series to absolutely converge and conditionally converge.

(j) You should know what it means for a series to diverge.

(k) You should know the definitions of geometric series, harmonic series, alternating series and p-series.

(l) You should know the definition of power series with base point center x_0 as a series of the form $\sum_{n=0}^{\infty} a_n(x-x_0)^n$.

(m) You should know what it means for a power series to converge at a point x or what it means for a power series to converge on an interval.

(n) You should know what the interval of convergence of a power series is.

(o) You should know what the radius of convergence R of a power series is.

(p) You should be able to recognize that the sum of a power series is a function (defined on the interval on which the series converges).

(q) Know what it means for a function to be represented by a power series.

(r) Given a function (which is infinitely differentiable at a point x_0), know how to define its Taylor series centered at x_0. This is the series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n = f(x_0) + \frac{f^{(1)}(x_0)}{1!}(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \cdots .$$

Major Results

You should thoroughly understand all of the major results we've discussed this semester; this includes all of the theorems and propositions we've discussed in class. This means, in particular, having a solid understanding of each theorem's hypothesis and, if I have discussed the necessity of a certain hypotheses, you should understand why. Here are some of the big results we've discussed so far.

1. You should know the (algebraic) laws of limits and how to use them.
2. You should know the squeeze theorem for sequences.
3. You should know the following theorem:

   Theorem A (Monotone Convergence Theorem)**.** *Every bounded monotonic sequence converges.*

4. You should know all of our results about geometric series.
5. You should know the (algebraic) laws of series.
6. You should know the following convergence/divergence test for series: If the sequence $\{a_n\}$ does not converge to zero, then the series $\sum_n a_n$ diverges.
7. You should know the integral test (and that it only applies to certain series with positive terms).
8. You should know the convergence results concerning p-series.
9. You should know the comparison test (and that it only applies to series with nonnegative terms).
10. You should know the alternating series test.
11. You should know the ratio test.
12. You should know the relationship between absolute convergence and convergence.
13. You should know the following theorem concerning the convergence of power series.

    Theorem B. *Consider a power series $\sum_n a_n(x - x_0)^n$. There are only three possibilities:*

    (a) The series converges only at the point $x = x_0$.

    (b) There exists a number $0 < R < \infty$ such that the series converges absolutely for all x in the interval $(x_0 - R, x_0 + R)$.

(c) The series converges absolutely for all real numbers x.

In the above theorem, R is called the radius of convergence of the power series. Note that the interval of convergence $(x_0 - R, x_0 + R)$ is the set of points x for which $|x - x_0| < R$. Remember, the theorem does not say what happens when $x = x_0 + R$ or $x = x_0 - R$ and, at these points, anything can happen.

14. You should know the result about differentiation and antidifferentiation of power series. You should know how differentiation and antidifferentiation affects convergence, i.e., the radius of convergence is the same but the behaviour at endpoints might be different.

Computation

In general, you should know how to work problems connected to each key concept discussed in the previous section. As I said before, if you understand all of the lecture material and all of the homework (both the "turn in" and "do not turn in" problems), are able to work quickly and efficiently, you should perform well on the exam. Here is an incomplete list of things we have done so far this semester:

1. Finding limits of sequences.
2. Using the definition of convergence (in terms of partial sums) to determine if a series converges.
3. You should know all of the computations related to geometric series.
4. You should know how to apply all of the tests for series (integral, alternating series, comparison, alternating series, ratio test, etc). Note: This is a lot of material and takes up a large portion of the computations you know how to do in this course.
5. Given a power series, you should know how to find the radius of convergence and the corresponding interval of convergence. Specifically, you should generally do this in three steps:
   (a) Use the ratio test to determine the radius of convergence and the open interval $(x_0 - R, x_0 + R)$ on which you will know the series absolutely converges.
   (b) Check the endpoints. Do this by plugging in $x = x_0 - R$ and $x = x_0 + R$ into the series and analyze your result as a numerical series. Use any tests you need.
   (c) Write down your interval of convergence.
6. Given a function f and a point x_0, you should be able to compute the the Taylor series of f.

Review Problems

The following problems will provide you with good practice for the midterm. If you want more problems then please feel free to ask.

Chapter 11 Summary (*p.602-603*) 1-34, 37-56, 59-62