This is the forth homework assignment for Math 122 and it is broken into two parts. The first part of the homework consists of exercises you should do (and I'll expect you to do) but you needn't turn in. As these exercises will not be graded, if you would like help with them or just want to make sure you're doing them correctly, you should (always) feel free to come to office hours (mine or those of the TAs). The second part is the part you are expected to turn in. More precisely, please complete all problems in Part 2, write up clear and thorough solutions for them (consistent with the directions given in the syllabus) and hand them in. Your write-ups are due on Wednesday, October 3rd at the beginning of class. As always, please come and see me early if you get stuck on any part of this assignment. I am here to help!

Part 1 (Do not turn in)

Exercise 1. *Please do Exercises #7, 23 from Section 11.4 of the textbook.*

Exercise 2. *Please do Exercises #5, 7, 9, 15, 17, 19, 33-38 from Section 11.5 of the textbook.*

Exercise 3. *Please do Exercises #5, 17, 51 from Section 11.6 of the textbook.*

Part 2 (Solutions for these problems are due in class on October 3rd)

Problem 1. *Determine whether the following series are absolutely convergent, conditionally convergent or neither.*

1. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k+1}{2^k}$
2. $\displaystyle\sum_{k=2}^{\infty}(-1)^{k}\ln\left(1+\frac{1}{k}\right)$
3. $\displaystyle\sum_{k=2}^{\infty}(-1)^{k}\frac{1}{\sqrt{k^2-1}}$
4. $\displaystyle\sum_{k=7}^{\infty}\frac{\cos(7k)}{7^k+k^7+\sin^7(7k)}$

Problem 2. *In each case, either give an example of a series which meets the following criteria, or explain why no such example can exist. If you use any theorems, results, or tests, be sure to mention them.*

(a) A series $\sum a_k$ such that $\sum a_k$ converges, but $\sum |a_k|$ diverges.

(b) A series $\sum a_k$ such that $\sum_{k=1}^{\infty} |a_k|$ converges, but $\sum_{k=1}^{\infty} a_k$ diverges.

(c) A series $\sum a_k$ such that $\sum a_k$ and $\sum |a_k|$ both converge.

(d) A series $\sum a_k$ such that $\sum a_k$ and $\sum |a_k|$ both diverge.

(e) A series which is both conditionally convergent and absolutely convergent.

(f) A series $\sum a_k$ such that $a_k \geq 0$ for all k and $\sum a_k$ converges conditionally.

Problem 3. *Consider the power series*

$$\sum_{n=2}^{\infty}\frac{(-2)^n}{n\ln(n)}(x+2)^n.$$

Perform a complete analysis of this power series. In doing so, find:

1. *All numbers x for which the series converges absolutely*
2. *All numbers x for which the series diverges*
3. *All number x for which the series converges conditionally*
4. *The interval of convergence*

In the course of your analysis, please make sure to identify which test(s) were used to make each conclusion.

Problem 4. *Let a and b be real numbers such that $a < b$.*

1. *Find a power series*
$$\sum_{k=0}^{\infty} a_k (x - x_0)^k$$
whose interval of convergence is (a, b).
2. *Find a power series*
$$\sum_{k=0}^{\infty} a_k (x - x_0)^k$$
whose interval of convergence is $[a, b]$.
3. *Find a power series*
$$\sum_{k=0}^{\infty} a_k (x - x_0)^k$$
whose interval of convergence is $[a, b)$.
4. *Find a power series*
$$\sum_{k=0}^{\infty} a_k (x - x_0)^k$$
whose interval of convergence is $(a, b]$.