This is the third homework assignment for Math 122 and it is broken into two parts. The first part of the homework consists of exercises you should do (and I'll expect you to do) but you needn't turn in. As these exercises will not be graded, if you would like help with them or just want to make sure you're doing them correctly, you should (always) feel free to come to office hours (mine or those of the TAs). The second part is the part you are expected to turn in. More precisely, please complete all problems in Part 2, write up clear and thorough solutions for them (consistent with the directions given in the syllabus) and hand them in. Your write-ups are due on Wednesday, September 26th at the beginning of class. As always, please come and see me early if you get stuck on any part of this assignment. I am here to help!

Part 1 (Do not turn in)

Exercise 1. *Please do Exercises 39 and 42 from Section 11.2 of the textbook.*

Exercise 2. *Please do Exercises 3, 11, 17, 19, 27 and 29 from Section 11.3 of the textbook. For these problems, you can ignore the directions concerning finding upper/lower bounds. Simply use an appropriate test to determine whether or not the given series converges. For Exercise 11, integration by parts might be useful.*

Exercise 3. *Please do Exercises 19, 23 and 25 from Section 11.4 of the textbook.*

Part 2 (Solutions for these problems are due in class on September 26th)

Problem 1. *Determine whether or not the following series converge. By using our results from class (and/or the textbook), justify your answer.*

1.
$$1+\frac{1}{1\cdot 3}+\frac{1}{1\cdot 3\cdot 5}+\frac{1}{1\cdot 3\cdot 5\cdot 7}+\cdots+\frac{1}{1\cdot 3\cdot 5\cdot 7\cdots(2k+1)}+\cdots$$

2.
$$\sum_{k=1}^{\infty}\frac{1}{100+2001\cdot k}$$

3.
$$\sum_{n=2}^{\infty}\frac{1}{n\ln n}$$

4.
$$\sum_{m=1}^{\infty}\frac{m}{4^m}$$

5.
$$\sum_{j=0}^{\infty}\frac{j!}{(j+2)!}$$

6.
$$\sum_{l=0}^{\infty}\frac{l!}{(2l)!}$$

Problem 2. *For which values of p, if any, do the following series converge? Justify your answer.*

1. $\displaystyle\sum_{k=2}^{\infty}\frac{1}{\ln(k)k^p}$

   Hint: one approach is to use the Comparison Test to determine the convergence/divergence of the series $\sum_{k=3}^{\infty}\frac{1}{\ln(k)k^p}$

2. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^p}$

3. $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{\ln(k)}{k^p}$

   Hint: you may find it useful to consider an appropriate function $f(x)$ and an application of L'Hopital's Rule.

Problem 3. *In this problem, we will consider what happens if we remove one of the requirements of the integral test.*

Theorem A (The Integral Test)**.** *Suppose that for all $x \geq 1$, the function $f(x)$ is continuous, non-negative, and decreasing. Then, the series*

$$\sum_{k=1}^{\infty} f(k)$$

converges if and only if the improper integral

$$\int_1^{\infty} f(x)\,dx$$

converges.

This theorem says that if the function $f(x)$ is continuous, positive, and decreasing, then the improper integral of the function behaves in the same way as the series $\sum_{k=1}^{\infty} f(k)$. In particular, if one of the two converges, so must the other; likewise, if one diverges, so must the other. [On Homework 2, you investigated one particular example of this, and related the divergence of the harmonic series $\sum_{k=1}^{\infty} \frac{1}{k}$ to the divergence of the integral $\int_1^{\infty} \frac{1}{x}\,dx$.]

1. *Let $f(x) = \cos(\pi(2x+1)) + 1$. Graph this function (use Wolfram Alpha or Desmos to get a nice-looking graph). By studying this graph, make some observations about the function $f(x)$.*

2. *Using the same function $f(x)$, consider the improper integral*

   $$\int_1^{\infty} f(x)\,dx.$$

   Does this improper integral converge? Explain your answer carefully.

3. *Again using the same function $f(x)$, consider the series*

   $$\sum_{k=1}^{\infty} f(k).$$

   Does this series converge?

4. *The previous two parts should show you that the integral $\int_1^{\infty} f(x)\,dx$ and the series $\sum_{k=1}^{\infty} f(k)$ behave very differently for this function $f(x)$. In view of your observations from Part 1, why is this not a contradiction to the Integral Test?*