MATH 122, F17
MIDDLEBURY COLLEGE

VERIFICATION OF FORMULA FOR S_n:

We have $S_n = \frac{b-a}{n}\left(f(x_1) + \dots + f(x_n)\right)$ ⊛

where

$$x_1 = a + \frac{(b-a)}{n}$$

$$x_2 = a + \frac{2}{n}(b-a)$$

$$\vdots$$

$$x_n = a + \frac{n}{n}(b-a)$$

FOR US: $a = 1$, $b = 2$, so we obtain

$$x_1 = 1 + \frac{1}{n} = \frac{n+1}{n}$$

$$x_2 = 1 + \frac{2}{n} = \frac{n+2}{n}$$

$$x_3 = 1 + \frac{3}{n} = \frac{n+3}{n}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$x_n = 1 + \frac{n}{n} = \frac{n+n}{n}$$

Hence,

$$f(x_1) = x_1 + 1 = \frac{n+1}{n} + 1 = \frac{2n+1}{n}$$

$$f(x_2) = x_2 + 1 = \frac{n+2}{n} + 1 = \frac{2n+2}{n}$$

$$f(x_3) = x_3 + 1 = \frac{n+3}{n} + 1 = \frac{2n+3}{n}$$

$$\vdots$$

$$f(x_{n-1}) = x_{n-1} + 1 = \frac{n+(n-1)}{n} + 1 = \frac{2n+(n-1)}{n}$$

$$f(x_n) = x_n + 1 = \frac{n+n}{n} + 1 = \frac{2n+n}{n}$$

Plugging back in to ⊛ above:

$$S_n = \frac{1}{n}\left(\frac{2n+1}{n} + \frac{2n+2}{n} + \dots + \frac{2n+(n-1)}{n} + \frac{2n+n}{n}\right).$$

QED.