



## THE BASICS

**Instructor:** George Melvin

**Contact:** [gmelvin@middlebury.edu](mailto:gmelvin@middlebury.edu)

**Office:** Warner Hall 312

**Office hours:** M 2–3pm, W 4–5pm, Th 9–11am

Alternatively you are welcome to chat with me when my office door is open and I am available, or by appointment.

**Course website:** <http://community.middlebury.edu/~gmelvin/current.html>

Announcements and handouts can be found at the course website. The course website is the primary resource for this course; please check frequently.

**Canvas & Piazza:** This course has a Canvas site accessible via Course Hub. Please ensure you can access the [piazza.com](https://piazza.com) forum using the ‘Piazza’ tab on the course Canvas site.

**Important dates:**

**3/8:** Examination I

**3/16:** Last day to drop course

**TBA:** Group Project

**4/12:** Examination II

**5/??:** Examination III

## THE COURSE

**Course description:** The theme of the course is the exact and approximate representation of real numbers and real-valued functions.

Initially we will be guided by the problem *How can we approximate, or specifically determine, area?* Approximating the area of planar regions will motivate an investigation of the essential analytic notions of *sequences* and *series*. Sequences will provide us with the correct framework to study compound interest leading to a development of the *exponential function*  $\exp(x)$  and its functional inverse, the *natural logarithm*  $\ln(x)$ . Further applications will include evaluations of limits, improper integrals, Newton’s method, rates of growth, and decimal approximations of irrational numbers.

The exponential function  $\exp(x)$  is defined via its *power series representation*. We will investigate further this fundamental analytic concept and consider how we may represent well-known functions by power series. This will lead us to Taylor’s Theorem and polynomial approximation of functions.

Further,  $\exp(x)$  is the solution to a *differential equation*. This leads to our final focus: *How can we obtain exact, or approximate, solutions to (first-order) differential equations?* Determining exact solutions will necessitate a development of our techniques of integration: integration by (trigonometric) substitution and by parts, and the method of partial fractions. We will apply these techniques to obtain solutions to linear/separable first-order differential equations and enlarge our definite integral toolbox to include computations of arc length, area of surfaces of revolution, and volume.

**Mathematical topics:** Brief review of the Riemann integral, antiderivatives, Fundamental Theorem of Calculus. Definition and properties of sequences, (non-)convergence and limits, convergence tests; definition and properties of series, alternating series, (non-)convergence of series and limits, absolute and conditional convergence. Improper integrals. Inverse functions: the exponential and natural logarithm; inverse trigonometric functions. Further techniques of integration: by substitution, by parts,

trigonometric substitution, partial fractions. Linear/separable first-order differential equations. Parametric curves; polar coordinates, polar curves; arc length. Cavalieri's principle and surface area, volume computations. Power series and radius of convergence, representation of functions by power series and their basic properties; Taylor series, Taylor's Remainder Theorem and polynomial approximation of functions.

**Prerequisites:** Math 122 is a continuation of the material covered in Math 121. If you have taken *AP Calculus AB*, or a course introducing differentiation, optimisation problems, integration and the Fundamental Theorem of Calculus, then you will be suitably prepared. If you are concerned about your preparation for this course then arrange to meet with me as soon as possible.

**Resources:** There is no required course textbook. Lecture notes will be supplied and posted at the course website. **You are warmly encouraged to make use of the following resources.**

The following recommended resources are supplementary to the lecture notes and will be placed on Reserve in the Davis Library. The books are ordered by increasing level of 'mathematical maturity'.

- *Calculus II*, Marsden & Weinstein
- *Single Variable Calculus*, Stewart (Ch. 7-11)
- *Calculus*, Spivak (Ch. 12, 18, 19, 20, 22-24)

Many other calculus textbooks can be found in the QA303.xxx section of the Davis Library, in the Math Department and in my office (available for consultation whenever I'm around). **Caution:** some textbooks with *calculus* in the title are more suitable for a course in *real analysis* (Math 0323).

Additionally, you can consult the following excellent online resources:

- *Community Calculus*, <https://www.whitman.edu/mathematics/multivariable/multivariable.pdf>
- *Kahn Academy*, <https://www.khanacademy.org/math/calculus-home>
- *Paul's Online Math Notes*, <http://tutorial.math.lamar.edu>

In line with the College Honor Code you are expected to properly attribute any additional resource you consult. If you have a question regarding the suitability of a resource, or appropriate attribution practices, then let me know.

## THE SKINNY

**Grading:** Your overall grade is given by the following prescription:

- Examination I (two hours) - 15% (March 8, 2018. Time/space location TBA.)
- Examination II (two hours) - 25% (April 12, 2018. Time/space location TBA.)
- Examination III (three hours)- 35% (Time/space location TBA.)
- Homework - 15%
- Project Work - 10%

I reserve the right to modify this prescription until no later than the end of the fifth week of the semester.

**Assignment of Grades:** The assignment of grades will be (roughly) as follows: let  $x$  be your final (weighted) score out of 100.

$x \geq 90$	<b>A</b>
$80 \leq x < 90$	<b>B</b>
$65 \leq x < 80$	<b>C</b>
$50 \leq x < 65$	<b>D</b>
$x < 50$	<b>F</b>

Scores that are near a boundary will be assigned an appropriate decoration  $\pm$ . The appendage of a decoration ( $\pm$ ) will be assigned at my discretion.

**Grading policy:** If you do better on Exam III than on Exam I (i.e. your score on Exam III, relative to the distribution of Exam III scores, improves from Exam I to Exam III) then your Exam I score will be disregarded and your Exam III score will constitute 50% of your overall grade.

**Examination policy:** You must take each Examination at the prescribed time. Known scheduling conflicts must be announced to me as soon as possible and additional arrangements will be made on a case-by-case basis. Failure to attend an Examination without excuse results in a failure for the Examination; there will be no make-up Examinations. Exceptions to this policy will only be granted in compelling circumstances.

**Homework:** Homework will be assigned on Monday, Wednesday, Friday and due for submission by 4pm on the day of the following MWF lecture. Homework will not be assigned/submitted on Thursday. You are **strongly** encouraged to work with your peers on homework. You are warmly encouraged to discuss problems with me. However, you should write up solutions *on your own* and *in your own words*.

The goal of homework in this class is to provide you with an opportunity to practice the skills learned in class. If you required further practice then take the initiative and make use of additional resources.

**Homework policy:** For each homework submission, I will ask the grader to grade five problems. Each problem will be worth 2 points and graded as follows:

- **0 points:** Illegible work; no justification (even if answer is correct);  $> 3$  minor errors or  $\geq 1$  major errors.
- **1 point:** Substantial progress made towards solution, but may have 2–3 minor errors or insufficient justification.
- **2 points:** Substantial progress made towards correct solution;  $\leq 1$  minor errors.

The lowest five homework scores will be dropped.

**Group project:** There is a required group project submission scheduled for the middle of the semester. The project is an opportunity for you to work in a group and investigate the historical development of an important concept/result related to the course material. Several project choices will be made available to you. Further details will be given before the end of Week 5 of the semester.

**Attendance:** Timely attendance of lectures is mandatory. If you expect regular class attendance to be a problem then let me know immediately and we can discuss your situation. In particular, this applies to athletics-explained absences. Changes to the attendance policy will only be granted in exceptional circumstances.

**Late Days:** You will be permitted *Late Days*<sup>TM</sup> that you can use throughout the semester. Late Days can be used to excuse late/absent attendance from class or late homework. Specific details can be found in the *Late Days* policy at the course website.

**Suggested collaboration policy:**

- You are strongly encouraged to collaborate with your peers. However, you should write up solutions *on your own* and *in your own words*.
- Any collaboration undertaken to obtain a solution on submitted work must be explicitly acknowledged; such acknowledgements will not be penalised. For example, write ‘*I worked with E. Noether, Archimedes, C. F. Gauss*’ on your submission.
- You are strongly encouraged to be an active participant in the [piazza.com](https://www.piazza.com) forum accessible via the course Canvas site.
- Other online forums (eg [math.stackexchange.com](https://math.stackexchange.com)) must not be utilised to complete or check your solutions to submitted work: consulting such forums is declared *cheating* in this class. You are reminded of your commitment to the College Honor Code.

**Extra credit:** There will be several opportunities made available to you to be awarded extra credit such as being awarded credit for challenge problem submissions, submitting short historical papers, and being creative with the course material and technology. Specific details on obtaining extra credit in this course will be provided at the course website and announced throughout the semester.

## THE IMPORTANT STUFF

**Classroom etiquette:** You are expected to be seated at your desk and ready to engage by the beginning of the class. You are expected to be courteous to your classmates and to help foster an inclusive, safe learning environment. Do not talk over each other; do not disregard someone's viewpoint; avoid the use of phrases like *'that's easy'*. Our learning environment will not be competitive. These remarks also apply to the [piazza.com](http://piazza.com) forum.

Mobile devices must be kept silent and non-vibrating throughout the duration of class. Use of your mobile device in-class is not permitted *unless otherwise specified*; if you are expecting a call then let me know. If you wish to use a laptop to take notes and/or document the class then you are free to do so. You must only use your computer as a note-taking device *unless otherwise specified*. Laptops should be silent throughout the class. Repeated violations of this policy will reflect negatively on your final score.

**Learning etiquette:** Be honest in your approach to learning. *Ask questions:* whenever you struggle with a topic, if an explanation is unclear, if notation is not defined; you will not be the only one with that question. *Direct questions:* to yourself, to your classmates, to me.

*Ask questions even if you think they are stupid.*

I have found that one of the most difficult tasks to undertake when learning is asking a *useful* question. You can help yourself by adopting the following 3RA strategy when asking questions:

- REFLECT: Ask yourself the question: does the question answer itself?
- WRITE: Write down your question down as a coherent sentence: does this help you see your way to a solution?
- REFORMULATE: Having constructed a coherent formulation of your question: can you ask a more pointed question?
- ASK: Ask your question!

Get to know your classmates and form study groups: discuss the material, work through problems, ask questions, help each other. If you understand something then challenge yourself by trying to explain your understanding to your peers. If someone struggles with your explanation then reformulate your argument: use examples, visual aids, simple analogies. The onus lies more greatly on the teacher to provide an adequate explanation than it does on the student to comprehend that explanation. If you are seeking explanation then *formulate your question as a coherent sentence*.

"What I cannot create, I do not understand"

*Richard Feynman, 1918-1988*

*Renowned physicist, educator and Nobel Laureate.*

**Drop-in guided study sessions:** The Center for Teaching, Learning and Research coordinates drop-in guided study sessions for calculus students every Sunday, Tuesday and Thursday, 7-9pm, in the Mathematics Department common area, 3<sup>rd</sup> floor Warner Hall. These study sessions are led by fellow student Calculus Gurus.

**Anti-discrimination commitment:** I am firmly committed to diversity and equality in all areas of campus life. In this class we will promote an anti-discriminatory environment where everyone feels safe and welcome. I recognize that discrimination can be direct or indirect and take place at both institutional and personal levels. I believe that such discrimination is unacceptable and I am committed to providing equality of opportunity for all by eliminating any and all discrimination, harassment, bullying, or victimization. The success of this policy relies on the support and understanding of everyone in this

class. We all have a responsibility not to be offensive to each other, or to participate in, or condone harassment or discrimination of any kind.

**Accommodations:** Students who have Letters of Accommodation in this class are encouraged to contact me as early in the semester as possible to ensure that such accommodations are implemented in a timely fashion. For those without Letters of Accommodation, assistance is available to eligible students through Student Accessibility Services. Please contact Jodi Litchfield or Courtney Cioffredi, the ADA Coordinators, for more information; all discussions will remain confidential.

- Jodi Litchfield ([litchfie@middlebury.edu](mailto:litchfie@middlebury.edu), or 802-443-5936),
- Courtney Cioffredi ([ccioffredi@middlebury.edu](mailto:ccioffredi@middlebury.edu), or 802-443-2169).

**Finally, a parting thought:** Please remember that MATHEMATICS IS DIFFICULT! But then again, getting admitted to Middlebury College is difficult and here you all are. We will strive towards a high level of rigor and it can be a struggle to wade through the mathematical marsh of complex concepts, technical tricks, and difficult definitions. However, if you are dedicated to your work, exercise your problem-solving abilities frequently, and talk about mathematics with your peers then I guarantee that you will be able to achieve your goals, and more, for this course.

### EXPECTATIONS & COMMITMENT PLEDGE

Our course is a shared experience. As such, we *all* play an important role. To help ensure a successful experience, I expect all of us, myself included, to commit to the following practices:

- **Be here.** Attend lectures, arrive on time, and stay for the duration of the class period.
- **Be prepared.** Complete preview activities and readings prior to attending each class.
- **Be present.** Plan to participate in class by both asking and answering questions, as well as by taking part in discussions and group activities.
- **Be proactive** in your understanding. Complete assignments regularly. Ask questions. Attend office hours regularly.
- **Be honorable.** Follow the Honor Code for all graded work in this course.
- **Be respectful** of yourself, your classmates, your instructor, and the classroom experience. Value yours and others' efforts and contributions to this class. Be mindful that we each arrive with different backgrounds and experiences. Your respectful presence and interactions with others are necessary to maintain a productive, safe, welcoming, and stimulating class environment. This respect should extend outside of our classroom to encompass our entire course experience, including assigned group work and homework collaboration.

I take these expectations very seriously. As such, I have signed the following commitment below and ask that all students in my courses sign and return the attached commitment as well.

*I understand the above expectations and commit to uphold them to the best of my ability through our learning experience.*

---

## MATH 122B, SPRING 2018. EXPECTATIONS & COMMITMENT PLEDGE

Our course is a shared experience. As such, we *all* play an important role. To help ensure a successful experience, I expect all of us, myself included, to commit to the following practices:

- **Be here.** Attend lectures, arrive on time, and stay for the duration of the class period.
- **Be prepared.** Complete preview activities and readings prior to attending each class.
- **Be present.** Plan to participate in class by both asking and answering questions, as well as by taking part in discussions and group activities.
- **Be proactive** in your understanding. Complete assignments regularly. Ask questions. Attend office hours regularly.
- **Be honorable.** Follow the Honor Code for all graded work in this course.
- **Be respectful** of yourself, your classmates, your instructor, and the classroom experience. Value yours and others' efforts and contributions to this class. Be mindful that we each arrive with different backgrounds and experiences. Your respectful presence and interactions with others are necessary to maintain a productive, safe, welcoming, and stimulating class environment. This respect should extend outside of our classroom to encompass our entire course experience, including assigned group work and homework collaboration.

I take these expectations very seriously. As such, I have signed the following commitment below and ask that all students in my courses sign and return the attached commitment as well.

*I understand the above expectations and commit to uphold them to the best of my ability through our learning experience.*

---