Middlebury College

Calculus II: Spring 2018

Homework

Due May 9, 6pm

Contact: gmelvin@middlebury.edu

Some thoughts and advice:

- You should expect to spend at least $1-2$ hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.
- You **are not allowed** to use any additional resources. If you are concerned then please ask.

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1. Determine whether the improper integral is convergent of divergent. If convergent, determine the limit.
   (a) $\int_1^\infty \frac{3x}{\sqrt{x^2-1}}dx$
   (b) $\int_0^2 \frac{3x}{2+x-x^2}dx$
   (c) $\int_{-2}^2 \frac{1}{4-x^2}dx$ (*Split the integral as a sum of two Type II integrals*)
   (d) $\int_{-\infty}^\infty |x|e^{-x^2}dx$
2. (a) For which p is the improper integral
   $$\int_1^\infty \frac{1}{x^p}dx$$
   convergent?
   (b) Use the Integral Test to show that $\sum_{n=1}^\infty \frac{1}{n^p}$ is convergent whenever $1 < p < 2$.
3. Let k be a constant. We are going to show that general solution to the differential equation
   $$\frac{dy}{dx} = ky$$
   is given by $y = Ae^{kx}$, with A a constant.
   (a) Show that ye^{kx} is a solution to the differential equation.

(b) Let $y = f(x)$ be a solution to the differential equation. Show that

$$\frac{d}{dx}\left(\frac{f(x)}{e^{kx}}\right) = 0$$

(c) Deduce that there is a constant A such that $f(x) = Ae^{kx}$.