MAR. 7

MATH 122B: HW ~~[illegible]~~

1) a) Let $a_n = \dfrac{n+5}{5n^2-2}$ $\quad b_n = \dfrac{1}{n}$

Then,

$$\frac{a_n}{b_n} = \frac{n+5}{5n^2-2} \cdot \frac{n}{1}$$

$$= \frac{n^2+5n}{5n^2-2}$$

$$= \frac{\cancel{n^2}}{\cancel{n^2}} \cdot \frac{(1+5/n)}{5-2/n^2} \xrightarrow{n\to\infty} \frac{1+0}{5-0}$$

$$= \frac{1}{5} > 0$$

Hence, since $\sum_{n=1}^{\infty} \frac{1}{n}$ divergent, the same is true of $\sum_{n=1}^{\infty} \frac{n+5}{5n^2-2}$, by LCT

b) Let $a_n = \dfrac{5^n}{7^n - 3^n}$, $\quad b_n = \left(\frac{5}{7}\right)^n$

$$\frac{a_n}{b_n} = \frac{\cancel{5^n}}{7^n-3^n} \cdot \frac{7^n}{\cancel{5^n}}$$

$$= \frac{7^n}{7^n-3^n} = \frac{\cancel{7^n}}{\cancel{7^n}} \quad \frac{1}{1-\left(\frac{3}{7}\right)^n}$$

$$\longrightarrow \frac{1}{1-0} = 1 > 0$$

Since $\sum_{n=1}^{\infty} \left(\frac{5}{7}\right)^n$ convergent, by Geom. Series Theorem, the same is true of $\sum_{n=1}^{\infty} \frac{5^n}{7^n-3^n}$, by LCT

c) Let $a_n = \dfrac{\sqrt{n^4+10}}{n^3+n^2}$, $b_n = \dfrac{1}{n}$.

$$\frac{a_n}{b_n} = \frac{\sqrt{n^4+10}}{n^3+n^2} \cdot \frac{n}{1}$$

$$= \frac{\sqrt{n^4(1+10/n^4)}}{n^2(n+1)} \cdot \frac{n}{1}$$

$$= \frac{\cancel{n^2}\sqrt{1+10/n^4}}{\cancel{n^2}(1+1/n)}$$

$$= \frac{\sqrt{1+10/n^4}}{1+1/n} \longrightarrow \frac{\sqrt{1+0}}{1+0} = 1 > 0$$

Hence, since $\sum\limits_{n=1}^{\infty} \frac{1}{n}$ divergent, the same is true of $\sum\limits_{n=1}^{\infty} \frac{\sqrt{n^4+10}}{n^3+n^2}$, by LCT

d) Let $a_n = \dfrac{n+10^n}{n+5^n}$, $b_n = 2^n$.

Then,

$$\frac{a_n}{b_n} = \frac{n+10^n}{(n+5^n)} \cdot \frac{1}{2^n}$$

$$= \frac{n+10^n}{2^n n+10^n} = \frac{\cancel{10^n}}{\cancel{10^n}} \cdot \frac{\left(\frac{n}{10^n}+1\right)}{\left(\frac{n}{5^n}+1\right)}$$

$$= \frac{\left(\frac{n}{10^n}+1\right)}{\left(\frac{n}{5^n}+1\right)} \longrightarrow \frac{0+1}{0+1} = 1 > 0$$

using FACT.

Hence, since $\sum_{n=1}^{\infty} 2^n$ divergent, by GST, same is true of $\sum_{n=1}^{\infty} \frac{n+10^n}{n+5^n}$ by LCT.

e) Let $a_n = \frac{1}{n^{1+1/n}}$ $b_n = \frac{1}{n}$

Then, $\frac{a_n}{b_n} = \frac{1}{n^{1+1/n}} \cdot \frac{n}{1}$

$$= \frac{n}{n \cdot n^{1/n}} = \frac{1}{n^{1/n}} \xrightarrow[\text{FACT}]{} \frac{1}{1} > 0$$

Hence, since $\sum \frac{1}{n}$ divergent, same true of $\sum \frac{1}{n^{1+1/n}}$, by LCT.

2a) Let $b_n = 1/n$. Then

$$\frac{a_n}{b_n} = \frac{a_n}{1/n} = n a_n \longrightarrow c > 0$$

Hence, since $\sum \frac{1}{n}$ divergent, same true of $\sum a_n$, by LCT

b) Let $b_n = \frac{1}{n^p}$. Then,

$$\frac{a_n}{b_n} = \frac{a_n}{1/n^p} = n^p a_n \longrightarrow c > 0.$$

Since $\sum \frac{1}{n^p}$ $\begin{Bmatrix} \text{convergent} \\ \text{divergent} \end{Bmatrix}$ if

$\begin{Bmatrix} p>1 \\ 0<p\leq 1 \end{Bmatrix}$ same holds for $\sum a_n$, by LCT.

Hence, $\sum a_n$ convergent ~~by~~ when $p>1$.

3) Let $a_n = \frac{n^p}{2+n^3}$, $b_n = \frac{1}{n^{3-p}}$

Then,

$$\frac{a_n}{b_n} = \frac{n^p}{2+n^3} \cdot \frac{n^{3-p}}{1}$$

$$= \frac{n^3}{2+n^3} = \frac{\cancel{n^3}}{\cancel{n^3}} \cdot \frac{1}{2/n^3+1}$$

$$= \frac{1}{2/n^3+1} \longrightarrow \frac{1}{0+1} = 1 > 0.$$

Hence, $\sum \frac{n^p}{2+n^3}$ convergent whenever $\sum \frac{1}{n^{3-p}}$ convergent, by LCT.

$\sum \frac{1}{n^{3-p}}$ convergent whenever $3-p>1$

$$\Rightarrow \boxed{2>p}.$$

4a) F

b) F

c) F

d) F

$$-\frac{1}{2n-1} - \frac{1}{2n}$$

$$= \frac{1}{4n^2-2n}$$

$\sum_{n=1}^{\infty} \frac{1}{4n^2-2n}$ convergent by LCT (compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$)