Calculus II: Spring 2018
Homework
Due March 7, 6pm
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Some thoughts and advice:

- You should expect to spend at least 1 – 2 hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words.*
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.
- You **are not allowed** to use any additional resources. If you are concerned then please ask.

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1. Use the Limit Comparison Test to determine whether the following series are convergent or divergent.
   (a) $\sum_{n=1}^{\infty} \frac{n+5}{5n^2-2}$
   (b) $\sum_{n=1}^{\infty} \frac{5^n}{7^n-3^n}$
   (c) $\sum_{n=1}^{\infty} \frac{\sqrt{n^4+10}}{n^3+n^2}$
   (d) $\sum_{n=1}^{\infty} \frac{n+10^n}{n+5^n}$ (*You may use the following* **fact***:* $\lim_{n\to\infty} \frac{n}{r^n} = 0$, for any $r > 1$)
   (e) $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$ (*You may use the following* **fact***:* $\lim_{n\to\infty} n^{1/n} = 1$)
2. (a) Let $\sum_{n=1}^{\infty} a_n$ be a series of positive terms (i.e. $a_n > 0$). Suppose that (na_n) is convergent and $\lim_{n\to\infty} na_n > 0$. Prove that $\sum_{n=1}^{\infty} a_n$ is divergent. (*Hint: determine an appropriate application of the Limit Comparison Test*)
   (b) Let $\sum_{n=1}^{\infty} a_n$ be a series of positive terms (i.e. $a_n > 0$). Let $p > 0$ be a real number so that $(n^p a_n)$ is convergent and $\lim_{n\to\infty} n^p a_n > 0$. For which values of p is $\sum_{n=1}^{\infty} a_n$ convergent? (*Hint: determine an appropriate application of the Limit Comparison Test*)
3. For what values of p does the series $\sum_{n=1}^{\infty} \frac{n^p}{2+n^3}$ converge?
4. True/False (no justification needed)
   (a) The series $\sum_{n=1}^{\infty} \frac{\sqrt{n^5+n^2}}{\sqrt[3]{n^7+n^3}}$ is convergent.
   (b) The series $\sum_{n=1}^{\infty} \frac{1}{2^n+n}$ is divergent.
   (c) There exists natural numbers c, d such that $\sum_{n=1}^{\infty} \frac{1}{cn+d}$ is convergent.
   (d) The series $\sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n}\right)$ is divergent.