Middlebury College

Calculus II: Spring 2018
Homework
Due March 16, 6pm

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Some thoughts and advice:

- You should expect to spend at least 1 – 2 hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.
- You **are not allowed** to use any additional resources. If you are concerned then please ask.

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1. Use mathematical induction to prove the following propositions.

   (a) $n^3 - n$ is divisible by 3, for every natural number n.

   (b) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, for every natural number n.

   (c) $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, for every natural number n.

   (d) $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

   (e) $\frac{1}{2^n - 1} < \frac{3}{2^n}$, for every natural number n. (*Recall this magical inequality from March 1 Lecture*)

2. For which non-negative integers n is $2n + 3 \leq 2^n$? Prove your answer using mathematical induction.

3. Consider the sequence (a_n) defined as follows:

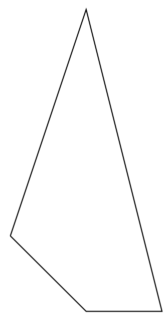
$$a_1 = 2, \quad a_{n+1} = \sqrt{6 + a_n}, \quad n \geq 2$$

   (a) Using mathematical induction show that $a_n < 3$, for every natural number n.

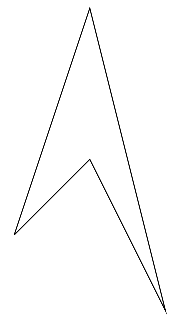
   (b) Using mathematical induction show that $a_n < a_{n+1}$, for every natural number n.

   (c) Show that the sequence (a_n) is convergent and determine $L = \lim_{n\to\infty} a_n$. (*Hint: you may use the fact that if* (b_n) *is convergent then* $\lim_{n\to\infty} b_{n+1} = \lim_{n\to\infty} b_n$.)

4. A **convex** n**-gon** is a polygon having n edges so that the angle between any two successive edges is less than π. For example, a convex 3-gon is a triangle.



convex 4-gon



not convex 4-gon

Using mathematical induction prove that the sum of the interior angles of a convex n-gon is equal to $(n-2)\pi$, for $n \geq 3$. (*Hint: a convex n-gon can be cut into an $(n-1)$-gon and a triangle*)

5. Explain why the following 'proof' is incorrect:

   Claim: All dogs have the same colour.

   Proof: Let $P(n)$ be the proposition '*all dogs in a set of n dogs have the same colour*'. Then, clearly $P(1)$ is true. Suppose that $P(n)$ is true, so that all dogs in any set of n dogs have the same colour. Consider a set of $n+1$ dogs and label them $1, 2, 3, \ldots, n+1$. Then, the first n dogs must have the same colour, by the inductive hypothesis. Similarly, the last n dogs must have the same colour. Since these two sets of dogs overlap, all $n+1$ dogs must have the same colour.