Calculus II: Spring 2018

Homework

Due $\sim \pi$, 6pm

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Some thoughts and advice:

- You should expect to spend at least 1 – 2 hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.
- You **are not allowed** to use any additional resources. If you are concerned then please ask.

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1. For the following series determine convergence/divergence using both the Ratio Test and the Root Test (if possible).

   (a) $\sum_{k=1}^{\infty} k\left(\frac{2}{3}\right)^k$

   (b) $\sum_{n=1}^{\infty} \frac{(-3)^n}{2n+1}$

   (c) $\sum_{m=1}^{\infty} \frac{m!}{(-100)^m}$

   (d) $\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n}}$

   (e) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

2. In this problem you will use series to show that *exponentials grow faster than polynomials* and *factorials grow faster than exponentials*. Let r be a real number, $|r| > 1$.

   (a) i. Show that $\sum_{n=1}^{\infty} \frac{n}{r^n}$ converges.

   ii. Deduce that $\lim_{n\to\infty} \frac{n}{r^n} = 0$.

   iii. Let k be a natural number. Use a similar approach to show that $\lim_{n\to\infty} \frac{n^k}{r^n} = 0$.

   (b) i. Show that $\sum_{n=1}^{\infty} \frac{r^n}{n!}$ converges.

   ii. Deduce that $\lim_{n\to\infty} \frac{r^n}{n!} = 0$.

3. A series $\sum_{n=1}^{\infty} a_n$ has terms defined by

$$a_1 = 1, \quad a_{n+1} = \frac{2+\cos(n)}{\sqrt{n}} a_n$$

Determine whether $\sum a_n$ converges or diverges.

4. True/False (no justification needed)

   (a) The series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+2n^2}$ is convergent, by the Ratio Test.

   (b) The series $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$ is convergent, by the Root Test.

   (c) There exists a divergent alternating series $\sum_{n=1}^{\infty}(-1)^n b_n$ for which $\sum_{n=1}^{\infty} b_n$ is convergent.

   (d) If $\sum_{n=1}^{\infty} a_n$ is divergent then $\sum_{n=1}^{\infty} |a_n|$ is divergent.