Middlebury College

Calculus II: Spring 2018
Homework
Due March 12, 6pm

Contact: gmelvin@middlebury.edu

Some thoughts and advice:

- You should expect to spend at least $1-2$ hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.
- You **are not allowed** to use any additional resources. If you are concerned then please ask.

---

1. Use the Alternating Series Test to show that the following series are convergent.
   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n!)^2}$
   (b) $\sum_{n=1}^{\infty} \frac{(-n)^n}{n^{3n}}$
   (c) $\sum_{n=1}^{\infty} \left(\frac{-2}{n}\right)^{3n}$
   (d) $\sum_{n=4}^{\infty} \frac{(-1)^n(1-n)}{3n-n^2}$
2. For each of the series above determine whether they are absolutely convergent or conditionally convergent.
3. Consider the series $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$, where
$$b_n = \begin{cases} \frac{1}{n^2}, & \text{if } n \text{ even}, \\ \frac{1}{n^3}, & \text{if } n \text{ odd}. \end{cases}$$
   (a) Explain why the Alternating Series Test does not apply to this series.
   (b) Show that $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ is convergent.
4. For which value of p is the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ convergent?
5. True/False (no justification needed)

(a) The series $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{2n+3}$ is convergent.

(b) The series $\sum_{n=1}^{\infty} \frac{(-n)^n}{n!}$ is divergent.

(c) Any alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$, with (b_n) bounded, is convergent.

(d) There exists a convergent alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ satisfying $\lim_{n\to\infty} b_n \neq 0$.