Middlebury College

Calculus II: Spring 2018

Homework

Due February 28, 4pm

Contact: `gmelvin@middlebury.edu`

Some thoughts and advice:

- You should expect to spend at least 1 – 2 hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.

- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.

- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.

- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.

- You **are not allowed** to use any additional resources. If you are concerned then please ask.

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1. Use a telescoping series argument to determine the limit of the following series.

   (a) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

   (b) $\sum_{n=1}^{\infty} \frac{1}{n(n+k)}$, where k is a natural number.

   (c) $\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2}$ (*Hint: use a partial fraction decomposition of the summand*)

   (d) $\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n-1)}$

2. Consider the series $\sum_{n=1}^{\infty} \frac{n+1}{n^2+2n}$. Explain carefully why you can't determine the limit of this series using the method of telescoping series. Does this necessarily imply that the series is divergent?

3. Determine whether the following series are convergent or divergent. Justify your answer.

   (a) $\sum_{n=1}^{\infty} \frac{n^2}{2n^2+5n+10}$

   (b) $\sum_{n=2}^{\infty} \frac{1}{n^2-1} + \frac{2^n}{3^{n+1}}$

   (c) $\sum_{n=1}^{\infty} \frac{3n^2-1}{n^2}$

   (d) $\sum_{n=1}^{\infty} \frac{3}{2n(n+2)} + \frac{5}{3n(n+3)}$

4. True/False (no justification needed)

   (a) There exists a divergent series $\sum_n a_n$ such that (a_n) is convergent.

   (b) If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ is convergent.

(c) If both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are divergent then $\sum_{n=1}^{\infty}(a_n + b_n)$ is divergent.

(d) If (a_n) is convergent then $\sum_{n=1}^{\infty} a_n$ is convergent.

5. Consider the curves $y = x^n$, $0 \leq x \leq 1$, $n = 0, 1, 2, 3, \ldots$. By considering the areas between curves, give a geometric demonstration of the fact that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$.