Middlebury College

Calculus II: Spring 2018

Homework

Due February 26, 4pm

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Some thoughts and advice:

- You should expect to spend at least 1 – 2 hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course piazza forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use piazza to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out khanacademy.org.
- You **are not allowed** to use any additional resources. If you are concerned then please ask.

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1. Use the Monotonic Bounded Theorem to show that the following sequences (a_n) are convergent.
   (a) $a_n = \frac{4}{(n+1)^3}$
   (b) $a_n = \frac{n+1}{5^n}$
   (c) $a_n = \frac{n}{n+3}$
   (d) $a_n = \frac{n}{2n+1}$

2. Define $n! \overset{def}{=} n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 2 \cdot 1$, when $n = 1, 2, 3, 4 \dots$. We say '*n factorial*' for the symbol $n!$. We adopt the convention that $0! \overset{def}{=} 1$.
   (a) Compute $n!$, for $n = 1, 2, 3, 4, 5, 6, 7, 8$. *n! groOOOOOOWWWS quickly*
   (b) Let $a_n = \frac{2^n}{n!}$. Write down a_1, a_2, a_3, a_4, a_5.
   (c) Suppose that (b_n) is a sequence satisfying $\frac{b_n}{b_{n+1}} \leq 1$, $n = 1, 2, 3, \dots$. Explain why (b_n) is increasing. Formulate a similar sufficient condition for a decreasing sequence.
   (d) Use the Monotonic Bounded Theorem to show that (a_n) is convergent.
   (e) Show that $0 < a_n \leq \frac{4}{n}$, for $n = 1, 2, 3, \dots$, and use the Squeeze Theorem to determine $\lim_{n\to\infty} a_n$.

3. (a) Define what it means for a series $\sum_{n=1}^{\infty} a_n$ to be convergent.

(b) Let (a_n) be a sequence with m^{th} partial sum

$$s_m = a_1 + \ldots + a_m = \frac{m^2+1}{3m^2+2m-1}$$

Does the series $\sum_{n=1}^{\infty} a_n$ converge? If so, what's its limit? If not, explain carefully why not.

(c) Let $|r| < 1$. By writing $\sum_{n=0}^{\infty} r^n = 1 + \sum_{n=1}^{\infty} r^n$, show that $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$.

(d) Let $|r| < 1$. Show that $\sum_{n=k}^{\infty} r^n = r^k + r^{k+1} + \ldots = \frac{r^k}{1-r}$.

4. True/False (no justification needed)

(a) If (a_n) is decreasing and bounded below by 0 then (a_n) is convergent and $\lim_{n\to\infty} a_n = 0$.

(b) The series $\sum_{n=1}^{\infty}(-1)^n$ is divergent.

(c) The series $\sum_{n=1}^{\infty} \frac{5^n}{3^{n+2}}$ is divergent.

(d) The series $\sum_{n=1}^{\infty} \frac{5^n}{3^{2n}}$ is divergent.

5. (a) Determine the limit of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{7^n}$.

(b) Determine the limit of the series $\sum_{n=1}^{\infty} \frac{3^n}{4^{n+2}}$.

(c) Use an appropriate series to determine a, b so that $0.712121212\cdots = \frac{a}{b}$.