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MATH 122: FEB 21. HW SOLUTION

1) a) $$a_n = \frac{3n^2 + 4n + 5}{2n^2 + 1}$$

$$= \frac{\cancel{n^2}\left(3 + \frac{4}{n} + \frac{5}{n^2}\right)}{\cancel{n^2}\left(2 + \frac{1}{n^2}\right)}$$

- Using limit laws we have $3 + \frac{4}{n} + \frac{5}{n^2}$ convergent with $\lim\left(3 + \frac{4}{n} + \frac{5}{n^2}\right) = 3 + 0 + 0 = 3$
- Using limit laws we have $2 + \frac{1}{n^2}$ convergent with $\lim\left(2 + \frac{1}{n^2}\right) = 2 + 0 = 2$

$$\Rightarrow \quad \lim a_n = \frac{3}{2}$$

b) $$a_n = \frac{n^2}{\sqrt{n^6 + 2n + 1}}$$

$$= \frac{n^2}{\sqrt{n^6\left(1 + \frac{2}{n^5} + \frac{1}{n^6}\right)}}$$

$$= \frac{n^2}{n^3} \cdot \frac{1}{\sqrt{1 + \frac{2}{n^5} + \frac{1}{n^6}}} = \frac{1}{n} \cdot \frac{1}{\sqrt{1 + \frac{2}{n^5} + \frac{1}{n^6}}}.$$

- The sequence $(\frac{1}{n})$ convergent with limit 0.
- Sequence $1 + \frac{2}{n^5} + \frac{1}{n^6}$ convergent with limit $1 + 0 + 0$, by Limit Laws.
$= 1$

- Using LL6, and fact that $f(x) = \sqrt{x}$ is continous on $[0, \infty)$, we get

$$\lim \sqrt{1 + \frac{2}{n^5} + \frac{1}{n^6}} = \sqrt{\lim \left(1 + \frac{2}{n^5} + \frac{1}{n^6}\right)}$$

$$= \sqrt{1} = 1.$$

Hence, sequence (a_n) convergent with limit $0 . 1 = \underline{\underline{0}}$ (note: , we could have stopped once we knew $\lim \frac{1}{n} = 0$)

c) Let $b_n = \frac{-1}{2\sqrt{n}}$, $c_n = \frac{1}{2\sqrt{n}}$

Then, using Limit Laws, $(b_n), (c_n)$ convergent with limit 0.

Also, $b_n \leq a_n \leq c_n$, for all n.

By Squeeze theorem, (a_n) convergent with limit 0.

d) Let $b_n = 2 - \frac{1}{n + \pi}$, $c_n = 2 + \frac{1}{n + \pi}$

The sequence $\frac{1}{n+\pi} = \frac{1}{n} \cdot \frac{1}{1+\frac{\pi}{n}}$

is convergent with limit $0 \cdot 1$

using limit laws.

$\Rightarrow$ (b_n), (c_n) convergent with limit

$2-0$, $2+0$

$=2$ $=2$

Also: $b_n \leq a_n \leq c_n$ for all n

$\Rightarrow$ By Squeeze Theorem, (a_n) convergent with limit 2.

2) a) $a_n = (-4)^{n-1}$

b) $a_n = 6n-3$

c) $a_n = \cos\left(\frac{\pi}{2} n\right)$

d) $a_n = 1 - \frac{1}{3^n}$

3a) $a_n = \frac{(-1)^n}{n}$; Let $b_n = -\frac{1}{n}$, $c_n = \frac{1}{n}$

Then, (b_n), (c_n) convergent and $\lim b_n = \lim c_n = 0$.

Also; $b_n \leq a_n \leq c_n$, for all n.

By Squeeze Theorem, (a_n) convergent (limit 0)

3 b) $(a_n) = (5, 5, 5, 5, \dots)$

- Constant sequences are convergent
- limit = 5

4a) F - eg $a_n = (-1)^n$

b) T

c) F - eg $a_n = b_n = n$

d) Same as (c)! OOOPS.

5a) Let $\varepsilon = 1$. Then, there is some k s.t

$$n \geq k \Rightarrow |a_n - L| < 1$$

But $|a_n - L| < 1$ is equivalent to inequalities:

$$L - 1 < a_n < L + 1.$$

b) Since $a_1, a_2, \dots, a_{k-1}$ is a finite set

we can always find some M

s.t. $M \geq \max(a_1, \dots, a_{k-1})$

Similarly, can find m s.t.

$$m \leq \min(a_1, \dots, a_{k-1})$$

$\Rightarrow$ $m \leq a_n \leq M$, for all $n = 1, 2, \dots, k-1$

5c) Spse (a_n) convergent with limit L.
Let k, M, m be as above.
Then, choose R s.t. $R = \max(M, L+1)$
and choose r s.t. $r = \min(m, L-1)$

Then, for every n,

$$r \leq a_n$$

and, for every n

$$a_n \leq R.$$

$\Rightarrow$ $r \leq a_n \leq R$, for every n

$\Rightarrow$ (a_n) bounded.