Calculus II: Spring 2018

Homework

Due February 21, 4pm

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Some thoughts and advice:

- You should expect to spend at least 1 – 2 hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.

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1. Each of the following sequences (a_n) converges. Determine the limit L and provide a careful justification for why (a_n) is convergent with limit L.
   (a) $a_n = \frac{3n^2+4n+5}{2n^2+1}$
   (b) $a_n = \frac{n^2}{\sqrt{n^6+2n+1}}$
   (c) $a_n = \frac{(-1)^n}{2\sqrt{n}}$
   (d) $a_n = 2 + \frac{(-1)^n}{n+\pi}$
2. Write down a formula for the n^{th} term of the sequence assuming that the pattern of the first few terms continues.
   (a) $(1, -4, 16, -64, 256, \ldots)$
   (b) $(3, 9, 15, 21, 27, \ldots)$
   (c) $(0, -1, 0, 1, 0, -1, 0, \ldots)$ (*Hint:* a trigonometric function might be useful here)
   (d) $(\frac{2}{3}, \frac{8}{9}, \frac{26}{27}, \frac{80}{81}, \ldots)$
3. (a) Give an example of a convergent sequence that is neither increasing nor decreasing. Make sure you explain why your example satisfies the required conditions.
   (b) Give an example of a convergent sequence (a_n) bounded below by 0 with limit $L = 5$. Make sure you explain why your example satisfies the required conditions.
4. True/False (no justification needed)

(a) Every divergent sequence is unbounded.

(b) If (a_n) is a sequence having positive terms (i.e. $a_n > 0$, for every n) and convergent with limit L then (b_n), where $b_n = \frac{a_n}{a_n+1}$, is convergent with limit $\frac{L}{L+1}$.

(c) If (a_n) is divergent and (b_n) is divergent then the sequence (c_n), where $c_n = \frac{a_n}{b_n}$, is divergent.

(d) If (a_n) is divergent and (b_n) is divergent then $\left(\frac{a_n}{b_n}\right)$ is divergent.

5. Let (a_n) be a convergent sequence with limit L.

(a) Explain why there is some natural number k with the property that $L-1 < a_n < L+1$, whenever $n \geq k$.

(b) Explain why it's possible to find real numbers m and M with the property that

$$m \leq a_n \leq M, \quad \text{whenever } 1 \leq n < k.$$

(c) Use the previous problems to provide argument establishing the following **Theorem**: *if (a_n) is a convergent sequence then (a_n) is bounded.*

The Theorem that you have just proved is equivalent to the following

Test for Divergence:

if (a_n) is unbounded then (a_n) is divergent.