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MATH 122: FEB. 19 HW SOLUTIONS

1a) $\cos(n\pi) = (-1)^n$

$\Rightarrow \quad (a_n) = (-1, 1, -1, 1, -1, 1, \dots)$

- Sequence is not increasing e.g. $a_3 < a_2$

  not decreasing eg $a_2 > a_1$

$\boxed{\text{NEITHER}}$

b) $$a_n - a_{n+1} = \left(2 + \frac{1}{n}\right) - \left(2 + \frac{1}{n+1}\right)$$

$$= \frac{1}{n} - \frac{1}{n+1} = \frac{1}{n(n+1)} > 0, \text{ for all } n$$

$\Rightarrow \quad a_n > a_{n+1}.$

$\boxed{\text{STRICTLY DEC.}}$

c) $a_1 = 1, \quad a_2 = 2 \cdot a_1 = 2, \quad a_3 = 2 \cdot a_2 = 4$ etc.

Since $a_n = 2a_{n-1} > a_{n-1}$, ~~(an)~~ strictly decreasing

$\Rightarrow \boxed{(a_n) \text{ STRICTLY INC.}}$

d) $$a_n - a_{n+1} = \left(2n + \frac{2}{5^n}\right) - \left(2(n+1) + \frac{2}{5^{n+1}}\right)$$

$$= -2 + \frac{2}{5^n}\left(1 - \frac{1}{5}\right)$$

$$= -2 + \frac{8}{5^{n+1}}$$

Claim: $-2 + \frac{8}{5^{n+1}} < 0$, for all n

Indeed, let $n \geq 1$. Then,

$$\frac{1}{4} > \frac{1}{5} > \frac{1}{5^2} > \frac{1}{5^3} > \cdots > \frac{1}{5^{n+1}}$$

ie $$\frac{2}{8} > \frac{1}{5^{n+1}}$$

$$\Rightarrow \quad 0 > \frac{8}{5^{n+1}} - 2 .$$

Hence, $\boxed{(a_n) \text{ STRICTLY DECREASING.}}$

e) $a_1 = 0 \quad a_2 = \frac{3}{2} \quad a_3 = \frac{8}{3}, \ldots$

$$a_n - a_{n+1} = n + \frac{(-1)^n}{n} - \left(n+1 + \frac{(-1)^{n+1}}{n+1}\right)$$

$$= -1 + (-1)^n \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= -1 + (-1)^n \left(\frac{1}{n(n+1)}\right)$$

$$= \begin{cases} -1 - \frac{1}{n(n+1)} & n \text{ odd} \\ -1 + \frac{1}{n(n+1)} & n \text{ even} \end{cases}$$

$$\leq \begin{cases} -1 & n \text{ odd} \\ -1 + \frac{1}{n} & n \text{ even (i.e } n = 2, 4, 6, \ldots) \end{cases}$$

$\Rightarrow$ Both cases we have are ≤ 0

$\Rightarrow (a_n)$ STRICTLY INCREASING.

lower

2) a) $\boxed{\text{Bounded}}$ between $-1, 1$ upper

b) • $2 + \frac{1}{n} > 2$, for any n

$\Rightarrow$ bounded below with lower bound 2

• Since $\frac{1}{n} \leq 1$, for all n;

$2 + \frac{1}{n} \leq 2 + 1 = 3$, for all n

$\Rightarrow$ bounded above with upper bound 3.

c) $\boxed{\text{Unbounded.}}$

$$
\begin{aligned}
a_n &= 2 a_{n-1} \\
&= \overbrace{2 \cdot 2}^{2} \cdot a_{n-2} \\
&= \overbrace{2 \cdot 2 \cdot 2}^{3} \cdot a_{n-3} \\
&\vdots \\
&= \overbrace{2 \cdot 2 \cdot 2 \cdots 2}^{n-1} \cdot a_{n-(n-1)} \\
&= 2^{n-1} \cdot a_1 = 2^{n-1}
\end{aligned}
$$

i.e 2^{n-1} grows without bound.

d) $a_n = 2n + \frac{2}{5^n} \geq 2n$, for all n.

Since $(2n)$ unbounded, same is true for (a_n) $\boxed{\text{UNBOUNDED}}$

e) $a_n = n + \frac{(-1)^n}{n} \geq n - 1$, since $\frac{(-1)^n}{n} \geq 1$

As $n-1$ unbounded same is true of (a_n). $\boxed{\text{UNBOUNDED}}$

3a) $a_n = (-1)^n$

b) $a_n = 1$, constant sequence

c) $a_n = (-1)^n . n$

d) $a_n = 0$, constant sequence.

4) a) F – $a_n = (-1)^n$

b) T

c) T

d) F – $a_n = (-1)^n n$

5) a) ~~tat~~ Since $\frac{1}{n^2} \leq \frac{1}{n}$, for all n,

and $\left(\frac{1}{n}\right)$ convergent with limit $L = 0$, we know that there is some N s.t.

$$n \geq N \implies \frac{1}{n} = \left|\frac{1}{n} - 0\right| < 3^{-3}$$

$$\implies \frac{1}{n^2} \leq \frac{1}{n} < 3^{-3}.$$

b) Let $\varepsilon > 0$. Then, since $(\frac{1}{n})$ convergent with limit $L = 0$, there's N s.t.

$$n \geq N \Rightarrow \frac{1}{n} = \left|\frac{1}{n} - 0\right| < \varepsilon$$

$$\Rightarrow \frac{1}{n^2} \leq \frac{1}{n} < \varepsilon.$$

c) $(\frac{1}{n^2})$ is convergent with limit $L = 0$.