Middlebury College

Calculus II: Spring 2018
Homework
Due February 19, 4pm

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Some thoughts and advice:

- You should expect to spend at least 1 – 2 hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.

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1. For the following sequences (a_n) determine whether they are **(strictly) increasing**, **(strictly) decreasing**, or **neither**. Provide justification for your claim.
   (a) $a_n = \cos(n\pi)$
   (b) $a_n = 2 + \frac{1}{n}$
   (c) $a_1 = 1$, $a_n = 2a_{n-1}$, for $n \geq 2$.
   (d) $a_n = 2n + \frac{2}{5^n}$
   (e) $a_n = n + \frac{(-1)^n}{n}$
2. Determine which of the above sequences are **bounded above**, **bounded below**, **bounded**, or **unbounded**. Provide justification and lower/upper bounds to support your claim.
3. (a) Give an example of a bounded below sequence that is neither increasing nor decreasing. Make sure you explain why your example satisfies the required conditions.
   (b) Give an example of a bounded sequence that is increasing but **not** strictly increasing. Make sure you explain why your example satisfies the required conditions.
   (c) Give an example of an unbounded sequence that is neither increasing nor decreasing. Make sure you explain why your example satisfies the required conditions.
   (d) Give an example of a sequence that is both increasing and decreasing. Make sure you explain why your example satisfies the required conditions.
4. True/False (no justification needed)

(a) Every divergent sequence is unbounded.

(b) Every increasing sequence is bounded below.

(c) There exists a bounded sequence that is neither increasing nor decreasing.

(d) Every sequence is either bounded below or bounded above.

5. **Fact:** The sequence (a_n), where $a_n = \frac{1}{n}$ is convergent with limit $L = 0$ (we will see this on February 19).

   (a) Using the Fact, explain why you can find a natural number N so that, if $n \geq N$ then $\frac{1}{n^2} < 3^{-3}$. (*Hint:* compare $\frac{1}{n^2}$ with $\frac{1}{n}$)

   (b) Let $\epsilon > 0$ be a real number. Using the Fact, explain why you can find a natural number N so that, if $n \geq N$ then $\frac{1}{n^2} < \epsilon$.

   (c) What does the previous problem tell you about the sequence (b_n), where $b_n = \frac{1}{n^2}$?