

MATH 122B : 2/16 HW SOLUTION

(1)

1a)

| | | | | | | | | |
|------|---|---|---|---|---|---|---|-----|
| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | ... |
| f(n) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | ... |
| T/F | F | T | F | T | F | T | F | ... |

(III)

b) (I) e.g. when $n > \sqrt{1000} = \frac{1}{\sqrt{0.001}}$
 then $|\frac{1}{n^2}| = \frac{1}{n^2} < \frac{1}{1000} = 0.001$.
 ie P holds when $n > \sqrt{0.001}$

c) (II) P only holds when
 $n < \sqrt{1000} = \frac{1}{\sqrt{0.001}}$

d)

| | | | | | | | |
|------|---|-----|---|-----|-----|-----|-----|
| n | 1 | 2 | 3 | 4 | 5 | 6 | ... |
| f(n) | 4 | 5/2 | 2 | 7/4 | 8/5 | 3/2 | ... |
| T/F | T | F | T | F | F | F | ... |

(II)

e) (I) similar to (b) : if $n > \sqrt{10^6}$
 then $\frac{1}{n^2} < \frac{1}{10^6} = 0.000001$.

(2)

2) P holds for $f(n)$ as $n \rightarrow \infty$:

|| there is a natural number N
so that, if $n \geq N$ then P holds
for $f(n)$

3) a) Lots of examples:

$$f(n) = -1 \quad (\text{constant})$$

$$f(n) = -1 + \frac{1}{n}$$

$$f(n) = -1 - \frac{1}{2^n} \quad \text{etc.}$$

b) Lots of examples:

$$P: '|y-1| < 0.1'$$

$$P: 'y < 1'$$

$$P: 'y > 0'$$

4) No: as soon as $n > 4$, $\frac{P_n}{7}$ is a fraction.

So, it's impossible to find N for which

$$n \geq N \Rightarrow \frac{P_n}{7} \text{ is an integer.}$$

- 5) a) F e.g. $P: 'y < 0.1'$
 $f(n) = \frac{1}{n}$ (3)
- b) F e.g. $P: 'y > 0'$
 $f(n) = -1,000,000,000,000,000,000,100$
 $+ n$
- c) T
- d) F : this is nonsense.