Middlebury College

Calculus II: Spring 2018
Homework
Due February 16, 4pm

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Some thoughts and advice:

- You should expect to spend at least $1-2$ hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.

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1. Determine whether $f(n)$ satisfies Condition (I), (II) or (III) for property P.
   (a) $f(n) = n$ and P : "y is an even number".
   (b) $f(n) = \frac{1}{n^2}$ and P : $|y| < 0.001$.
   (c) $f(n) = \frac{1}{n^2}$ and P : $|y| \geq 0.001$.
   (d) $f(n) = \frac{3}{n} + 1$ and P : "y is an integer".
   (e) $f(n) = \frac{1}{n^2}$ and P : $|y| < 0.000001$.
2. Let $f(n)$ be a real-valued function, P be a property of real numbers. Write down what it means for P to hold for $f(n)$ as $n \to \infty$.
3. (a) Consider the property P : $|y+1| < \frac{1}{5}$. Determine a real-valued function $f(n)$ (n is a natural number) so that P holds for $f(n)$ as $n \to \infty$.
   (b) Let $f(n) = 1 - \frac{1}{2^n}$. Give an example of a property P that holds for $f(n)$ as $n \to \infty$
4. Let $f(n) = \frac{p_n}{7}$, where p_n is the n^{th} prime number, and let P : "y is an integer". Is it possible to find N so that, for every $n \geq N$, property P holds for $f(n)$? Justify your answer.
5. Let $f(n)$ be a real-valued function, P be a property of real numbers. Suppose that P holds for $f(n)$ as $n \to \infty$. **True/False:**
   (a) property P holds for $f(n)$, for any n.
   (b) property P holds for $f(n)$, $n \geq 1,000,000,000,000,000,000,000,000$.
   (c) there is a really large natural number a for which property P holds for $f(n)$, whenever $n \geq a$.
   (d) property P holds for $f(n)$ when $n = \infty$.