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February 12 Lecture

In this exercise you will investigate the construction of the *Koch snowflake* and try to determine its area.

We are going to describe a *recursive* procedure to define a planar region K known as the Koch snowflake. Each stage of the recursion is called an *iteration*. At the start of the recursion, the $n = 0$ iteration, we define $K(0)$ to be an equilateral triangle (say the sides all have length 1 metre).

The next iteration of the recursive process creates the *snowflake* $K(1)$ by altering **each perimeter line segment** of the original triangle $K(0)$ as follows:

(i) Divide the line segment into three segments of equal length.

(ii) Draw an equilateral triangle that has the middle segment from the previous step as its base and points outward.

(iii) Remove the line segment that is the base of this newly created triangle.

Having completed the above steps you have constructed the $n = 1$ snowflake $K(1)$.
Useful formula: you will need the following formula

$$\text{area of equil. triangle with side length } a \text{ metres} = \frac{\sqrt{3}}{4}a^2 \text{ metres}^2$$

1. Follow the above procedure to construct the snowflake $K(1)$. Draw your snowflake below.

2. Determine the area of $K(0)$ and $K(1)$

   Area of $K(0)$ =

   Area of $K(1)$ =

3. Repeat steps (i)-(iii) for **each perimeter line segment of** $K(1)$ above to complete the second iteration of the recursive process. Once completed you have created the snowflake $K(2)$.

   Draw the snowflake $K(2)$ below. Determine the area of $K(2)$.

   Area of $K(2)$ =

4. (SPOT THE PATTERN!) Having completed two iterations of the recursive process you have created two snowflakes $K(1), K(2)$. We could continue and perform the third iteration to create a third snowflake $K(3)$ by applying the steps (i)-(iii) to each perimeter line segment of $K(2)$ (although, depending on the size of your original triangle, this may be difficult to draw!). We are interested in determining the area of $K(3)$ *using the power of our minds!* Can you spot any patterns from your previous work that would help you predict the area of $K(3)$?

5. Predict the area that we add on to the area of $K(2)$ to obtain the area of $K(3)$.

6. Predict the area we would add on to the area of $K(n)$ (the snowflake created by the n^{th} iteration of our process) to obtain the area of $K(n+1)$.

7. Use your formula to predict the area of $K(4)$.

8. Express the total area of $K(4)$ as a sum of five terms. (*Hint: the first term should be the area of the original triangle* $K(0)$)

9. As n gets bigger and bigger, you could follow our process to write down a formula for the area of $K(n)$ as a sum of $n+1$ terms (this is a *George-given truth*!). Circle the following statements that you believe are true.

- **as n gets very large the area of $K(n)$ gets very large, without bound**

- **there is some large n for which the area of $K(n)$ is** ≥ 100

- **for any n, the area of $K(n)$ is** ≤ 2