Middlebury College

Calculus II: Spring 2018

Contact: gmelvin@middlebury.edu

Practice Examination II

Instructions:

- You *must* attempt Problem 1.
- Please attempt at least three of Problems 2,3,4,5.
- If you attempt all five problems then your final score will be the sum of your score for Problem 1 and the scores for the three remaining problems receiving the highest number points.
- Calculators are not permitted.

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1. (10 points) True/False:

   (a) $\int_1^{\exp(2x)} \frac{1}{t} dt = 2 \int_1^x \frac{1}{t} dt$

   (b) The function $f(x) = 3 - \sin(x+1)$, $-1 \leq x \leq 2$, has an inverse function.

   (c) There exists a power series $\sum_{n=0}^{\infty} c_n (x-3)^n$ that converges at $x = -4$ and diverges at $x = 5$.

   (d) Let $f(x)$ be an infinitely differentiable function with associated Tayloer series (centred at $c = 2$)
   $$\sum_{n=0}^{\infty} \frac{5^n}{n} (x-2)^n.$$
   Then, $f^{(10)}(2) = \frac{5^{10}}{9!}$.

   (e) Let $\sum_{n=0}^{\infty} c_n x^n$ be a power series with interval of convergence $[-2, 2)$. Then, the radius of convergence of $\sum_{n=0}^{\infty} c_n (x-1)^n$ is $R = 1$.

2. Determine the interval of convergence of the following power series.

   (a)
   $$\sum_{n=0}^{\infty} \frac{2^n}{2n+1} (x-3)^n$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{n!}{(2n)!} (x+1)^n$$

3. Consider the function
   $$f(x) = \sum_{n=0}^{\infty} \frac{(x+2)^n}{2^n (n+2)}$$

   (a) Determine the domain A of $f(x)$.

   (b) What is $f''(-2)$?

   (c) Give a function $g(x)$ with domain A satisfying $\frac{d}{dx} g(x) = f(x)$.

4. Using induction show that

$$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\ldots+\frac{1}{(2n-1)\times(2n+1)}=\frac{n}{2n+1},$$

for any natural number n.

5. (a) Determine the associated Taylor series (centred at $c=5$) of $f(x)=\frac{1}{x}$.

   (b) Suppose $f(x)$ is a function with associated Taylor series (centred at $c=0$)

$$\sum_{n=0}^{\infty}\frac{x^n}{n(2n+1)}$$

   Determine the associated Taylor series (centred at $c=0$) of $f'(x)$.