MATH 122 : ~~3~~ 4/6 HOMEWORK

1a) $$f(x) = \frac{d}{dx}\left(\frac{1}{1-2x}\right)$$

$$= \frac{d}{dx}\left(1 + 2x + (2x)^2 + (2x)^3 + \cdots\right)$$

$$= 2 + 8x + 24x^2 + 64x^3 + \cdots$$

b) $$f(x) = x \cdot \frac{d}{dx}\left(\frac{1}{1-x}\right)$$

$$= x \cdot \frac{d}{dx}\left(1 + x + x^2 + x^3 + x^4 + \cdots\right)$$

$$= x \cdot \left(1 + 2x + 3x^2 + 4x^3 + \cdots\right)$$

$$= x + 2x^2 + 3x^3 + 4x^4 + \cdots$$

$$= \sum_{n=1}^{\infty} n x^n$$

c) $$\ln 5-x = -\int \frac{1}{5-x}\,dx$$

$$= -\int\left(\frac{1}{5}\left(1 + \frac{x}{5} + \frac{x^2}{25} + \frac{x^3}{125} + \cdots\right)\right)$$

$$= -\left(\frac{1}{5}x + \frac{x^2}{2 \cdot 25} + \frac{x^3}{3 \cdot 125} + \frac{x^4}{4 \cdot 625} + \cdots\right) + C$$

Note when $x = 0$ $\quad C = \ln(5)$

$$\Rightarrow \ln(5-x) = \ln(5) - \frac{1}{5}x - \frac{x^2}{50} - \frac{x^3}{375} - \cdots$$

d) $f(x) = (1-x) \cdot \frac{d}{dx}\left(\frac{-1}{1+x}\right)$

$= (1-x) \cdot \left[\frac{d}{dx}\left(-(1-x+x^2-x^3+x^4-\cdots)\right)\right]$

$= (1-x)\left(1-2x+3x^2-4x^3+\cdots\right)$

$= 1-2x+3x^2-4x^3+\cdots$

$\quad -x+2x^2-3x^3+4x^4 \cdots$

$= 1-3x+5x^2-7x^3+9x^4 \cdots$

2 a) $c_1 = \frac{1}{2}$, $c_2 = -\frac{1}{8}$, $c_3 = \frac{3}{48} = \frac{1}{16}$

$c_4 = \frac{-15}{384} = \frac{-5}{128}$, $c_5 = \frac{105}{32 \times 120}$

b) $\left|\frac{c_n}{c_{n+1}}\right| = \left| \frac{1\cdot(-1)\cdot(-3)\cdot(-5)\cdots(3-2n)}{2^n \cdot n!} \cdot \frac{2^{n+1}\cdot(n+1)!}{1\cdot(-3)\cdots(3-2n)\cdot(3-2(n+1))} \right|$

$= \left|\frac{2(n+1)}{3-2n-2}\right| = \left|\frac{2(n+1)}{1-2n}\right|$

$= \frac{2(n+1)}{2n-1}$, since $n \geq 1$.

c) $$R = \lim \left| \frac{c_n}{c_{n+1}} \right|$$

$$= \lim \frac{2(n+1)}{2n-1} = 1.$$

d) $$\cancel{\sqrt{1+x}} = \cancel{\sqrt{1+x}} = \cancel{1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \tfrac{1}{16}x^3 \ldots}$$

$$\sqrt{2} = \sqrt{1+1} = 1 + \frac{1}{2} - \frac{1}{8} + \frac{1}{16} - \frac{5}{128} \cdots$$

3a) $$\frac{d}{dx} f(x) = \frac{1}{xy} \cdot y = \frac{1}{x},$$ by chain rule.

b) Since $\frac{d}{dx} \ln(x) = \frac{d}{dx} f(x)$

there is some C s.t.

$$f(x) = \ln(x) + C.$$

c) $$\ln(y) = f(1) = \ln(1) + C = C$$

$$\Rightarrow \quad C = \ln(y).$$

$$\Rightarrow \quad f(x) = \ln(xy) = \ln(x) + \ln(y).$$