Middlebury College

Calculus II: Spring 2018

Homework

Due April 6, 6pm

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Some thoughts and advice:

- You should expect to spend at least 1 – 2 hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.
- You **are not allowed** to use any additional resources. If you are concerned then please ask.

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1. For the given function $f(x)$, determine a power series representation and state for which x the representation is valid.

   (a) $f(x) = \frac{2}{(1-2x)^2}$

   (b) $f(x) = \frac{x}{(1-x)^2}$

   (c) $f(x) = \ln(5-x)$

   (d) $f(x) = \frac{1-x}{(1+x)^2}$

2. For $n = 1, 2, 3, \ldots$, define

$$c_n = \frac{1 \cdot (-1) \cdot (-3) \cdot \cdots \cdot (3-2n)}{2^n n!}$$

   (a) Write down c_n, $n = 1, 2, 3, 4, 5$.

   (b) Show that

$$\left|\frac{c_n}{c_{n+1}}\right| = \frac{2(n+1)}{2n-1}$$

   (c) Deduce that the radius of convergence of the power series $1 + \sum_{n=1}^{\infty} c_n x^n$ is $R = 1$. In particular, the power series converges absolutely whenever $|x| < 1$.

(d) The first few terms of the power series $1 + \sum_{n=1}^{\infty} c_n x^n$ are

$$1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \dots \qquad (*)$$

Fact: whenever $-1 < x < 1$,

$$\sqrt{1+x} = 1 + \sum_{n=1}^{\infty} c_n x^n$$

We will see this in a few lectures time (you can also check by multiplying $(*)$ by itself and seeing it equals $1 + x$).
Use this Fact to give a series whose limit is $\sqrt{2}$.

3. The natural logarithm function $\ln(x)$, $x > 0$, is defined to be

$$\ln(x) = \int_1^x \frac{1}{t} dt$$

(a) Let $y > 0$. Define $f(x) = \ln(xy)$, $x > 0$. Show that $\frac{d}{dx} f(x) = \frac{1}{x}$.

(b) Explain why there exists a constant C such that $f(x) = \ln(x) + C$.

(c) Show that $C = \ln(y)$ and deduce that

$$\ln(xy) = \ln(x) + \ln(y), \quad x, y > 0$$