Middlebury College

Calculus II: Spring 2018
Homework
Due April 4, 6pm

Contact: gmelvin@middlebury.edu

Some thoughts and advice:

- You should expect to spend at least $1-2$ hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.
- You **are not allowed** to use any additional resources. If you are concerned then please ask.

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1. Show that
$$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$$
Here $\arctan(x)$, $-\infty < x < \infty$, is the inverse function of tan.

2. Determine the interval of convergence for the following power series.
   (a) $\sum_{n=1}^{\infty} \frac{(x-3)}{n+1}$
   (b) $\sum_{n=0}^{\infty} \frac{10^n x^n}{n^3}$
   (c) $\sum_{n=0}^{\infty} \frac{(-x)^n}{n^2 2^n}$
   (d) $\sum_{n=0}^{\infty} n(x+2)^n$
   (e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$

3. Suppose that $\sum_{n=1}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series? (*Hint: what is the centre? what must be true of the interval of convergence?*)
   (a) $\sum_{n=1}^{\infty} c_n$
   (b) $\sum_{n=1}^{\infty} c_n 8^n$
   (c) $\sum_{n=1}^{\infty} c_n (-3)^{-n}$
   (d) $\sum_{n=1}^{\infty} (-1)^n c_n 7^n$

4. Give a power series representation for the following functions and determine for which x the representations equal the given function.

   (a) $f(x) = \frac{2}{1-3x}$

   (b) $f(x) = \frac{1}{2+3x}$

   (c) $f(x) = \frac{1}{1+4x^2}$

5. True/False. No justification required.

   (a) If the power series $\sum c_n x^n$ converges at $x = 1$ then it converges at $x = -1/2$.

   (b) There exists a power series $\sum_{n=0}^{\infty} c_n(x-1)^n$ with interval of convergence $(-1, 3]$.

   (c) There exists a power series $\sum_{n=0}^{\infty} c_n(x+2)^n$ with interval of convergence $(1, 2]$.

   (d) Suppose $\sum b_n x^n$ has interval of convergence $(-1, 1)$. Then, $\sum b_n 2^n$ is divergent.