MATH 122 : HW 4/27

1a) $x^2 + 3x + 1 = x^2 + 3x + 9 - 9 + 1$

$= (x+3)^2 - 8$

REDUCIBLE

b) $2x^2 + 3x + 5$

$= 2\left(x^2 + \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right) + 5$

$= 2\left(x + \frac{3}{4}\right)^2 + \frac{31}{8}$ IRRED.

c) $-3x^2 + x + 2$

$= -3\left(x^2 - \frac{1}{3}x + \frac{1}{36} - \frac{1}{36}\right) + 2$

$= -3\left(x - \frac{1}{6}\right)^2 + \frac{25}{12}$ RED.

d) $3x^2 + x + 2$

$= 3\left(x + \frac{1}{6}\right)^2 + \cancel{\frac{23}{12}}\ \frac{23}{12}$ IRRED

e) $\cancel{3(x^2+}$ $5x^2 + 2$ IRRED.

2a) $$\frac{3x+1}{x^3+4x} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$\Rightarrow \quad 3x+1 = A(x^2+4) + x(Bx+C)$$

<u>Input</u>	<u>LHS</u>		<u>RHS</u>	
$x=0$	1	$=$	$4A$	$\Rightarrow A = 1/4$
$x=1$	4	$=$	$\frac{5}{4} + B + C$	
$x=-1$	-2	$=$	$\frac{5}{4} + B - C$	

$$\frac{11}{4} = B + C \quad (1)$$

$$-\frac{13}{4} = B - C \quad (2)$$

$(1) + (2)$: $\quad -\frac{1}{2} = 2B \quad \Rightarrow \quad B = -1/4$

$(1) \Rightarrow \quad \frac{11}{4} = -1/4 + C \quad \Rightarrow \quad C = 3.$

$$\int \frac{3x+1}{x^3+4x}\,dx = \int \frac{1}{4}\cdot\frac{1}{x} + -\frac{1}{4}\cdot\frac{x-12}{x^2+4}\,dx$$

$$\int \frac{x-12}{x^2+4}\,dx = \int \frac{x}{x^2+4}\,dx - 12\int\frac{dx}{x^2+4}$$

$$= \frac{1}{2}\ln|x^2+4| - \frac{12}{2}\arctan\left(\frac{x}{2}\right)$$

use sub. $\uparrow$ $v = x^2+4$

$$\Rightarrow \int \frac{3x+1}{x^3+4x}\,dx = \frac{1}{4}\ln|x| - \frac{1}{8}\ln|x^2+4| + \frac{3}{2}\arctan\left(\frac{x}{2}\right) + C$$

b) $$x^2 + 2x + 5 = (x+1)^2 + 4$$

$$\int \frac{x}{x^2+2x+5}\,dx = \int \frac{x}{(x+1)^2+4}\,dx$$

Let $u = x+1$ $\quad x = u-1$

$du = dx$

$$= \int \frac{u-1}{u^2+4}\,du$$

$$= \int \frac{u}{u^2+4}\,du - \int \frac{1}{u^2+4}\,du$$

$$= \frac{1}{2}\ln|u^2+4| - \frac{1}{2}\arctan\left(\frac{u}{2}\right) + C$$

$$= \frac{1}{2}\ln|(x+1)^2+4| - \frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + C.$$

c) $$x^2 + 4x + 7 = (x+2)^2 + 3$$

$$\int \frac{x+3}{x^2+4x+7}\,dx = \int \frac{x+3}{(x+2)^2+3}\,dx$$

$u = x+2$ $\quad x+3 = u+1$

$du = dx$

$$= \int \frac{u+1}{u^2+3}\,du$$

$$= \int \frac{u}{u^2+3}\,du + \int \frac{1}{u^2+3}\,du$$

$$= \frac{1}{2} \ln |u^2+3| + \frac{1}{\sqrt{3}} \arctan\left(\frac{u}{\sqrt{3}}\right) + C$$

$$= \frac{1}{2} \ln |(x+2)^2+3| + \frac{1}{\sqrt{3}} \arctan\left(\frac{x+2}{\sqrt{3}}\right) + C$$

d)

$$\frac{x^2-29x+5}{(x-4)^2(x^2+3)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{Cx+D}{x^2+3}$$

$$\Rightarrow \quad x^2-29x+5 = A(x-4)(x^2+3) + B(x^2+3) + (x-4)^2(Cx+D)$$

Input	LHS
$x=4$	$\cancel{1449}$ $-95 = \cancel{B}\ 19B \Rightarrow B=-5$
$x=0$	$5 = -12A - 15 + 16D$ ①
$x=1$	$-23 = -12A - 20 + 9(C+D)$ ②
$x=-1$	$35 = -20A - 20 + 25(-C+D)$ $\cancel{③}$
	$7 = -4A - 4 + 5(-C+D)$ ③

① - ② : $\overset{28}{\cancel{18}} = 0 + 5 + 16D - 9C - 9D$

$$\Rightarrow \boxed{23 = 7D - 9C.} \quad ④$$

① - 3×③ : $-16 = 0 - 3 + 16D - 15(-C+D)$

$$\Rightarrow \boxed{-13 = \cancel{1}D + 15C} \quad ⑤.$$

④ - 7×⑤ : $114 = -114C \Rightarrow C = -1$

④ $\Rightarrow$ $23 = 7D + 9 \Rightarrow D = 2$

① $\Rightarrow$ $5 = -12A - 15 + 32 \Rightarrow A = 1$

$$\int \frac{x^2-29x+5}{(x-4)^2(x^2+3)}dx = \int \frac{1}{x-4} - \frac{5}{(x-4)^2} + \frac{-x+2}{x^2+3}\, dx$$

$$= \ln|x-4| + \frac{5}{x-4} - \frac{1}{2}\ln|x^2+3| + \frac{2}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + C.$$

3) $$\int \frac{\sin(x)}{\cos^2(x)+5\cos(x)+6}\, dx$$

$u = \cos(x)$

$du = -\sin(x)\, dx$

$$= -\int \frac{1}{u^2+5u+6}\, du$$

$$= -\int \frac{1}{(u+3)(u+2)}\, du$$

$$\frac{1}{(u+3)(u+2)} = \frac{A}{u+3} + \frac{B}{u+2}$$

$\Rightarrow \quad 1 = A(u+2) + B(u+3)$

$\Rightarrow$

Input	LHS		RHS
$u=-2$	1	$=$	B
$u=-3$	1	$=$	$-A$

$$\int \frac{1}{(u+3)(u+2)}\, du = \int -\frac{1}{u+3} + \frac{1}{u+2}\, du$$

$$= -\ln|u+3| + \ln|u+2| + C$$

$$= -\ln|\cos(x)+3| + \ln|\cos(x)+2| + C.$$

4) $$\sec(x) = \frac{1}{\cos(x)} = \frac{\cos(x)}{\cos^2(x)}$$

$$= \frac{\cos(x)}{1-\sin^2(x)}$$

b) $$\int \sec(x)\,dx = \int \frac{\cos(x)}{1-\sin^2(x)}\,dx$$

$u = \sin(x)$

$du = \cos(x)\,dx$

$$= \int \frac{1}{1-u^2}\,du$$

$$\frac{1}{1-u^2} = \frac{A}{1-u} + \frac{B}{1+u}$$

$$\Rightarrow \quad 1 = A(1+u) + B(1-u)$$

Input	LHS	RHS
$u=1$	$1 = 2A$	$A = 1/2$
$u=-1$	$1 = 2B$	$B = 1/2$

$$\int \frac{1}{1-u^2}\,du = \int \frac{1}{2}\left(\frac{1}{1-u} + \frac{1}{1+u}\right)du$$

$$= \frac{1}{2}\left(-\ln|1-u| + \ln|1+u|\right) + C.$$

$$= \frac{1}{2}\left(-\ln|1-\sin(x)| + \ln|1+\sin(x)|\right) + C.$$

d) $$\frac{1}{2}\left(-\ln|1-\sin(x)| + \ln|1+\sin(x)|\right) + C$$

$$= \cancel{\frac{1}{2}\ln|(1-\sin(x))(1+\sin(x))|}$$

$$= \frac{1}{2}\ln\left|\frac{1+\sin(x)}{1-\sin(x)}\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{(1+\sin(x))^2}{1-\sin^2(x)}\right| + C$$

$$= \ln\left|\sqrt{\frac{(1+\sin(x))^2}{\cos^2(x)}}\right| + C$$

$$= \ln\left|\frac{1+\sin(x)}{\cos(x)}\right| + C$$

$$= \ln|\sec(x)+\tan(x)| + C.$$