Middlebury College

Calculus II: Spring 2018
Homework
Due April 23, 6pm

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Some thoughts and advice:

- You should expect to spend at least $1-2$ hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.
- You **are not allowed** to use any additional resources. If you are concerned then please ask.

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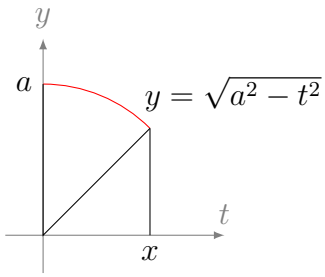
1. (a) Using inverse trigonometric substitution, show that

$$\int_0^x \sqrt{a^2-t^2}dt = \frac{a^2}{2}\arcsin\left(\frac{x}{a}\right) + \frac{1}{2}x\sqrt{a^2-x^2}$$

   Recall: if $F(x)$ is an antiderivative of $f(x)$ then $\int_a^b f(x)dx = F(b)-F(a)$.

   (b) Use the figure to give a geometric interpretation of the equation above:



2. Determine a partial fraction decomposition of the following rational functions.

a.
$$\frac{5x+1}{(x-1)(2x+1)}$$

b.
$$\frac{2}{x^2+3x+2}$$

c.
$$\frac{3x-4}{x^2-x-6}$$

d.

$$\frac{2x^2}{x^3+2x^2+x}$$

e.

$$\frac{x+2}{(x^2-1)^2}$$

f.

$$\frac{x+b}{x^2-a^2}$$

3. Determine the following antiderivative problems:

a.

$$\int \frac{5x+1}{(x-1)(2x+1)}dx$$

b.

$$\int \frac{2x-1}{x^2+5x+6}dx$$

c.

$$\int \frac{3x^3}{x^2-4x+4}dx$$

d.

$$\int \frac{1}{(x^2-x)^2}dx$$

e.

$$\int \frac{1}{x^3-3x+2}dx$$

f.

$$\int \frac{x^4}{x^3-2x^2-x+2}dx$$

For e., f., you will need to factorise the denominator by finding a root of the polynomial.