MATH 122: HW 4/18

1a) $\int \cos(x)\cos^4(x)\sin^2(x)\,dx$

$$= \int \cos(x)\left(1-\sin^2(x)\right)^2 \sin^2(x)\,dx$$

$u = \sin(x)$
$du = \cos(x)\,dx$

$$= \int (1-u^2)^2 u^2\,du$$

$$= \int u^2 - 2u^4 + u^6\,du$$

$$= \frac{u^3}{3} - \frac{2}{5}u^5 + \frac{u^7}{7} + C$$

$$= \frac{\sin(x)^3}{3} - \frac{2}{5}\sin(x)^5 + \frac{\sin(x)^7}{7} + C$$

b) $\int \frac{12x^3}{3x^4+1}\,dx$

$u = 3x^4 + 1$
$du = 12x^3\,dx$

$$= \int \frac{1}{u}\,du = \ln|u| + C$$

$$= \ln|3x^4+1| + C$$

c) $\int \frac{1}{\sqrt{3x-1}}\,dx$

$u = 3x - 1$
$du = 3dx$

$$= \frac{1}{3}\int \frac{1}{\sqrt{u}}\,du = \frac{2}{3}\sqrt{u} + C$$

$$= 2/3\sqrt{3x-1} + C.$$

d) $\int \frac{2e^x}{e^x+1}\,dx$

$u = e^x + 1$

$du = e^x\,dx$

$= \int \frac{2\,du}{u} = 2\ln|u| + C$

$= 2\ln|e^x+1| + C$

e) $\int \frac{1}{\sqrt{x}(x+1)}\,dx$

$u = \sqrt{x}$

$du = \frac{2}{2}\frac{1}{\sqrt{x}}\,dx$

$= \int \frac{2}{u^2+1}\,du$

$= 2\arctan(u) + C$

$= 2\arctan(\sqrt{x}) + C$

f) $\int \frac{1}{x}\frac{1}{\ln(x)}\,dx$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln(\ln(x)) + C.$

$$\cos^2(x) = \tfrac{1}{2}(1+\cos(2x))$$
$$\sin^2(x) = \tfrac{1}{2}(1-\cos(2x))$$

2) $$\int \cos^4(x)\sin^4(x)\,dx$$

$$= \int \left(\tfrac{1}{2}(1+\cos(2x))\right)^2 \left(\tfrac{1}{2}(1-\cos(2x))\right)^2 dx$$

$$= \frac{1}{16}\int \left(1-\cos^2(2x)\right)^2 dx$$

$$= \frac{1}{16}\int 1 - 2\cos^2(2x) + \cos^4(2x)\,dx$$

$$= \frac{1}{16}\int 1 - (1+\cos(4x)) + \left(\tfrac{1}{2}(1+\cos(4x))\right)^2 dx$$

$$= \frac{1}{16}\int -\cos(4x) + \frac{1}{4}\left(1 + 2\cos(4x) + \cos^2(4x)\right) dx$$

$$= \frac{1}{16}\int \frac{1}{4} - \frac{1}{2}\cos(4x) + \frac{1}{8}(1+\cos(8x))\,dx$$

$$= \frac{1}{128}\int 3 - 4\cos(4x) + \cos(8x)\,dx$$

$$= \frac{1}{128}\left(3x - \sin(4x) + \frac{\sin(8x)}{8}\right) + C.$$

3) b) $$\int \tan^2(x)\,dx = \int \sec^2(x) - 1\,dx$$

$$= \tan(x) - x + C$$

$$\int \sec^2(x)\,dx = \tan(x) + C$$

a) $$\frac{d}{dx}\frac{\sin(x)}{\cos(x)} = \frac{\cos(x)\cdot\cos(x) + \sin(x)\sin(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)} = \sec^2(x).$$

$$\frac{d}{dx}\frac{1}{\cos(x)} = -\frac{1}{(\cos(x))^2}\cdot(-\sin(x))$$

$$= \frac{\sin(x)}{\cos(x)}\cdot\frac{1}{\cos(x)}$$

$$= \tan(x)\cdot\sec(x).$$

c) $$\int \tan(x)\sec^2(x)\,dx$$

$u = \tan(x)$
$du = \sec^2(x)\,dx$

$$= \int u\,du = \frac{1}{2}u^2 + C$$

$$= \frac{1}{2}\tan(x)^2 + C.$$

d) $$\int \tan^3(x)\sec^4(x)\,dx$$

$$= \int \tan^3(x)\left(\tan^2(x)+1\right)\sec^2(x)\,dx,$$

since $\sec^2(x) = \tan^2(x) + 1$

$u = \tan(x)$
$du = \sec^2(x)\,dx$

$$= \int u^3(u^2+1)\,du$$

$$= \frac{u^6}{6} + \frac{u^4}{4} + C$$

$$= \frac{\tan(x)^6}{6} + \frac{\tan(x)^4}{4} + C.$$

e) $$\int \tan^4(x)\sec^3(x)\,dx$$

$$= \int \frac{\sin^4(x)}{\cos^7(x)}\,dx = \int \frac{\sin(x)}{\cos^7(x)}\cdot\sin^3(x)\,dx$$

$$g' = \frac{\sin(x)}{\cos^7(x)} \qquad f = \sin^3(x)$$

$$g = \frac{1}{6}\frac{1}{\cos^6(x)} \qquad f' = 3\sin^2(x)\cos(x)$$

$$= \frac{1}{6}\frac{\sin^3(x)}{\cos^6(x)} - \frac{1}{2}\int \frac{\sin(x)}{\cos^5(x)} \cdot \sin(x)\,dx$$

$$g' = \frac{\sin(x)}{\cos^5(x)} \qquad f = \sin(x)$$

$$g = \frac{1}{4}\frac{1}{\cos^4(x)} \qquad f' = \cos(x)$$

$$= \frac{1}{6}\frac{\sin^3(x)}{\cos^6(x)} - \frac{1}{2}\left[\frac{1}{4}\frac{\sin(x)}{\cos^4(x)} - \frac{1}{4}\int \sec^3(x)\,dx\right]$$

$$= \frac{1}{6}\frac{\sin^3(x)}{\cos^6(x)} - \frac{1}{8}\frac{\sin(x)}{\cos^4(x)} + \frac{1}{8}\cdot\left(\frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x)+\tan(x)|\right) + C.$$

f) $\int \sec^6(x)\,dx$

$$= \int (\tan^2(x)+1)^2 \sec^2(x)\,dx$$

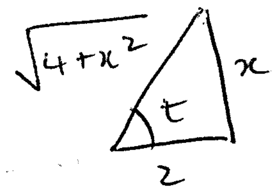
$$u = \tan(x)$$
$$du = \sec^2(x)\,dx$$

$$= \int (u^2+1)^2\,du$$

$$= \frac{u^5}{5} + \frac{2}{3}u^3 + u + C$$

$$= \frac{\tan^5(x)}{5} + \frac{2}{3}\tan^3(x) + \tan(x) + C.$$

4a) $x = 2\tan(t)$



$\frac{dx}{dt} = 2\sec^2(t)$

$$\int \frac{1}{(4+x^2)^{3/2}}\,dx = \int \frac{1}{(4+4\tan^2(t))^{3/2}} \cdot \sec^2(t)\,dt$$

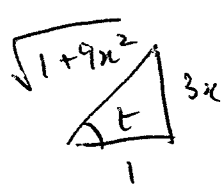
$$= \int \frac{1}{4^{3/2}} \cdot \frac{1}{\sec(t)}\,dt$$

$$= \frac{1}{8} \cdot \int \cos(t)\,dt$$

$$= \frac{1}{8}\sin(t) + C.$$

$$= \frac{1}{8}\,\frac{x}{\sqrt{4+x^2}} + C.$$

b) $3x = \tan(t)$



$\frac{dx}{dt} = \frac{1}{3}\sec^2(t)$

$$\int \frac{1}{(1+9x^2)^2}\,dx = \int \frac{1}{(1+\tan^2(t))^2}\cdot\frac{1}{3}\sec^2(t)\,dt$$

$$= \frac{1}{3}\int \frac{1}{\sec^2(t)}\,dt$$

$$= \frac{1}{3}\int \cos^2(t)\,dt$$

$$= \frac{1}{6}\int 1 + \cos(2t)\,dt$$

$$= \frac{1}{6}\left(t + \frac{\sin(2t)}{2}\right) + C$$

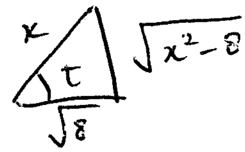
$$= \frac{1}{6}\left(t + \sin(t)\cos(t)\right) + C$$

$$= \frac{1}{6}\left(\arctan(3x) + \frac{3x}{\sqrt{1+9x^2}} \cdot \frac{1}{\sqrt{1+9x^2}}\right) + C$$

$$= \frac{1}{6}\left(\arctan(3x) + \frac{3x}{1+9x^2}\right) + C.$$

c) $x = \sqrt{8}\sec(t)$

$$\frac{dx}{dt} = \sqrt{8}\sec(t)\tan(t)$$



$$\int \frac{1}{\sqrt{x^2-8}}\,dx = \int \frac{1}{\sqrt{8\sec^2(t)-8}} \cdot \sqrt{8}\sec(t)\tan(t)\,dt$$

$$= \int \sec(t)\,dt$$

$$= \ln|\sec(t) + \tan(t)| + C$$

$$= \ln\left|\frac{x}{\sqrt{8}} + \frac{\sqrt{x^2-8}}{\sqrt{8}}\right| + C.$$