Middlebury College

Calculus II: Spring 2018
Homework
Due April 16, 6pm
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Some thoughts and advice:

- You should expect to spend at least $1-2$ hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site), come to office hours, or send me an email. However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets. **This will make your life easier!** You can use `piazza` to arrange meet-ups. However, you must write your solutions *on your own* and *in your own words*.
- If you would like more practice then there are (hundreds of) problems in the supplementary course textbooks mentioned in the syllabus, or you can check out `khanacademy.org`.
- You **are not allowed** to use any additional resources. If you are concerned then please ask.

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1. The Taylor series (centred at $c=0$) associated to $\cos(x)$ is

$$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\dots$$

   (a) Using Taylor's Theorem, show that $\cos(x)$ equals its Taylor series for $-10\leq x\leq 10$.

   (b) Let $d>0$. Show that $\cos(x)$ equals its Taylor series for $-d\leq x\leq d$.

   (c) Deduce that

$$\cos(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\dots,\quad \text{for any } x$$

   Fact: $\cos(x)$ is equal to its associated Taylor series (centred at *arbitrary* c), for any x.

   (d) Use the 6^{th} degree Taylor polynomial $T_6(x)$ to approximate $\cos(1)$.

2. Evaluate the following limits. *For c. use an appropriate power series centred at* $c=\pi$

   a. $$\lim_{x\to 0}\frac{\cos(x)-1}{x^2}$$

   b. $$\lim_{x\to 0}\frac{x}{e^{2x}-1}$$

   c. $$\lim_{x\to \pi}\frac{1+\cos(x)}{(x-\pi)^2}$$

3. Using integration by parts determine the following antiderivative problems.

a.

$$\int xe^{2x}dx$$

b.

$$\int x^2 \sin(3x)dx$$

c.

$$\int x(x+2)e^x dx$$

d.

$$\int (\ln(x))^2 dx$$

e.

$$\int (x+1)\ln(x)dx$$

f.

$$\int e^x \cos(x)dx$$

g.

$$\int \arctan(x)dx$$

h.

$$\int \frac{\ln(x)}{\sqrt{x}}dx$$