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Sigma notation

Let r be a natural number. Let $a_1, \dots, a_r$ be a collection of real numbers. We define

$$\sum_{i=1}^{r} a_i \stackrel{def}{=} a_1 + a_2 + \dots + a_{r-1} + a_r.$$

This notation is called *sigma notation* - the symbol Σ is the (capitalised) Greek letter '*sigma*'.

Basic properties

1. (*Independence of index of summation*) The notation does not depend on the *summation index* i appearing in $\sum_{i=1}^{r} a_i$, in the sense that

$$\sum_{i=1}^{r} a_i = \sum_{j=1}^{r} a_j = \sum_{\bullet=1}^{r} a_\bullet = \sum_{\Delta=1}^{r} a_\Delta$$

However, the index of summation should not be an already defined symbol. For example,

$$\sum_{r=1}^{r} a_r$$

does not make sense.

2. (*Bounds of summation*) For any $1 \leq p \leq q \leq r$ we write

$$\sum_{i=p}^{q} a_i = a_p + a_{p+1} + \dots + a_{q-1} + a_q.$$

In particular, if $p = q$ then we have

$$\sum_{i=p}^{p} a_i = a_p.$$

3. (*Additive properties*) Let $b_1, \dots, b_r$ be another collection of r real numbers, c a constant. Then,

$$\sum_{i=1}^{r} (a_i \pm b_i) = \sum_{i=1}^{r} a_i \pm \sum_{i=1}^{r} b_i, \qquad \text{and} \qquad \sum_{i=1}^{r} c a_i = c \sum_{i=1}^{r} a_i$$

Is s is a natural number

4. For any $1 \leq p \leq r$ we can split up sigma notation as follows:

$$\sum_{i=1}^{r} a_i = \sum_{i=1}^{p} a_i + \sum_{i=p+1}^{r} a_i.$$

It's important to have the index of summation in the latter sum begin at $p + 1$.