Middlebury College

Calculus II: Fall 2017
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Math 122 Course Review: Things to Know

Sequences & Series

1. Convergence, limit of a sequence: definition.
2. How to show convergence of a sequence (a_n):
   (a) Limit Laws: careful, can only use limit laws if a given sequence can be realised as an expression involving *known convergent sequences* e.g. constant sequences, $(\frac{1}{n^k})$, (x^n)
   (b) Squeeze Theorem
   (c) Monotonic+Bounded Theorem: to show *increasing* must show $a_n \leq a_{n+1}$, **for every** n: can do this *directly* or using *induction*. Similar considerations for decreasing.
3. Convergence, limit of a series: definition in terms of sequence of partial sums.
4. How to show convergence/divergence of a series $\sum a_n$:
   (a) Test for divergence
   (b) Geometric series: Use Geometric Series Theorem: $\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}$, whenever $-1 < r < 1$.
   (c) Series of positive terms: Direct Comparison Test, Limit Comparison Test.
   (d) Alternating series test
   (e) Ratio/root tests

Inverse Functions, exp and log

1. Inverse functions: what are they? Horizontal line test.
2. Definition of $\exp(x)$; basic properties; $\exp(x) = e^x$.
3. Definition of $\log(x)$ as a definite integral; $\log(x)$ as the inverse of $\exp(x)$; basic properties.

Induction

1. Proof by induction: practice, practice, practice!

Integration

1. Antiderivative problem: statement of problem. Don't forget '$+C$'!
2. Method of substitution
3. Method of integration by parts: what to do if $\log(x)$, $\arctan(x)$, $\arcsin(x)$ appear?
4. Method of inverse trigonometric substitution: completing the square
5. Method of partial fractions

6. Arc length & surface area of surface of revolution
7. Improper integrals: type I/II. Definition as a limit. Look out for 'bad points'.

Differential Equations

1. Growth & decay equation
2. Separable equations

> Given a separable differential equation
>
> $$\frac{dy}{dx} = g(x)h(y)$$
>
> (a) Solve the antiderivative problems
>
> $$\int \frac{1}{h(y)} dy = \int g(x)dx$$
>
> (b) Solve for y, if possible.

3. Linear equations:

> Given a linear first-order differential equation
>
> $$y' + P(x)y = Q(x)$$
>
> (1) Compute $\int P(x)dx$ (no constant of integration required)
>
> (2) Let $I(x) = \exp\left(\int P(x)dx\right)$.
>
> (3) Solve the antiderivative problem on the right hand side of the equation
>
> $$I(x)y = \int Q(x)I(x)dx$$
>
> (5) Rearrange for y, if possible.

4. Initial-value problems: use a given condition to get rid of the constant of integration.

Power series

1. Basics: definition of power series; centre. Interval of convergence: using Ratio/Root Test to determine interval of convergence. *Checking endpoints!*
2. Representing functions by power series: the *Calculus game* i.e. using known series (e.g. geometric series) to obtain power series representations.
3. Taylor series: definition, Taylor polynomials; Taylor Theorem/Inequality
4. Using power series to determine limits of series.