Middlebury College

# Calculus II: Fall 2017

Review Problem Set

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## Pre-calculus Cobweb Blaster

The following problems are intended to refresh your knowledge of some precalculus topics.

1. If $m = -1$, $n = 2$, find the value of $m^2 - 2mn - n^2$.
2. What is the slope of the line $3y + 4x = 2$?
3. For which values of $x$ is $3x + 2 > 0$?
4. Given the formula ${}_{r,s}\Delta_{t,u} = ru + s - t$, find ${}_{1,3}\Delta_{2,4}$.
5. For which values of $x$ is $3x^2 - 2x - 1 > 0$?
6. Solve for $x$: $2x^2 + 8x - 11 = 0$.
7. Let $f(x) = \frac{2}{x} - x^2$. Find $f(1), f(-1), f(r), f(2-b)$.
8. Sketch the graph of $h(y) = y^2 - y - 2$.
9. State the domain of the function $g(x) = \frac{3x^2+2x-8}{(2x^2-x)}$.
10. For what values of $a$ is $3a + 2 > 2a - 8$?
11. Where does the graph of $f(x) = 3x - 2$ intersect the graph of $g(x) = x^2$?
12. State the domain of the function $r(t) = \frac{1}{\cos(t^2)-1}$.
13. Given the formula $h(2x - 1) = x^2$, find $h(1), h(3)$, and $h(b)$.
14. Given the formula $j(i^2) = i^2$, find the value of $j(1), j(2), j(4)$.
15. Given the formulae $x(z) = y(2z)$ and $y(w) = 2w + 1$, find $x(3)$.
16. For each $\theta = \frac{\pi}{2}, \frac{2\pi}{3}, \frac{7\pi}{6}$ compute $\cos\theta, \tan\theta, \sec\theta, \csc^2\theta$.
17. Compute $2\cos^2\theta + 2\sin^2\theta$, where $\theta = \frac{2\pi}{13}$.

## Functions, Limits & Continuity Cobweb Blaster

1. Suppose that $f$ has domain $A$ and $g$ has domain $B$.
   (a) What is the domain of $f + g$?
   (b) What is the domain of $fg$?
   (c) What is the domain of $f/g$?
2. For which $x$ are the following functions discontinuous? At which of these values $x$ is $f$ continuous from the right, from the left, or neither? (*Hint: what is the the graph of* $f$)

(a) $f(x) = \begin{cases} 1+x^2, & \text{if } x \le 0, \\ 2-x, & \text{if } 0 < x \le 2, \\ (x-2)^2, & \text{if } x > 2. \end{cases}$

(b) $f(x) = \begin{cases} x+2, & \text{if } x < 0, \\ 2x^2, & \text{if } 0 \le x \le 1, \\ 2-x, & \text{if } x > 1. \end{cases}$

3. True/False:

   (a) If $f$ is a function then $f(s+t) = f(s) + f(t)$.

   (b) If $f(s) = f(t)$ then $s = t$.

   (c) Let $x, y$ be in the domain of $f$. If $x < y$ and $f$ is increasing then $f(x) < f(y)$.

   (d) A vertical line intersects the graph of $f$ at most once.

   (e) A vertical line intersects the graph of $f$ exactly once.

   (f) If $x$ is any real number then $x = \sqrt{x^2}$.

   (g) $\lim_{x\to 1}\left(\frac{2}{x-4} - \frac{3x}{x+5}\right) = \lim_{x\to 1}\frac{2}{x-4} - \lim_{x\to 1}\frac{3x}{x+5}$.

   (h) If $f(1) > 0$ and $f(3) < 0$ then there exists some $1 < c < 3$ such that $f(c) = 0$.

   (i) If $f$ is continuous at $x = 5$ and $f(5) = 2$ and $f(4) = 3$ then $\lim_{x\to 2} f(4x^2 - 11) = 2$.

   (j) If $f$ is continuous at $x = a$ then $|f|$ is continuous at $x = a$.

   (k) If $|f|$ is continuous at $x = a$ then $f$ is continuous at $x = a$.

## Derivative Cobweb Blaster

1. Compute the following derivatives:

   1. $\frac{d}{dx}(3x+1)$ 2. $\frac{d}{dx}(3x^2)$ 3. $\frac{d}{d\theta}2\cos(\theta^2)$ 4. $\frac{d}{dy}\left(\frac{y-3}{y^3}\right)$ 5. $\frac{d}{dx}\left(\sqrt{x} - \frac{1}{2x^2}\right)$

   6. $\frac{d}{ds}(s^2 e^{\cos(s)})$ 7. $\frac{d}{d\Delta}\tan(\Delta)$ 8. $\frac{d}{dt}\left(\frac{t^2-t}{t^3+2t}\right)$ 9. $\frac{d}{dx}\left(\frac{x^3\sqrt{x}}{x^{-2/3}}\right)$ 10. $\frac{d}{d\bullet}\left(\bullet(\sin(2\bullet^2) + \frac{1}{\bullet})\right)$

2. (a) Give the definition of the derivative $f'(a)$.

   (b) Find a function $f$ and a number $a$ such that

$$\lim_{h\to 0}\frac{(2+h)^6 - 64}{h} = f'(a).$$

3. True or False:

   (a) If $f$ is continuous at $a$ then $f$ is differentiable at $a$.

   (b) If $f$ is differentiable then $\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$.

   (c) If $f$ is differentiable then $\frac{d}{dx}f(\sqrt{x}) = \frac{f'(x)}{2\sqrt{x}}$.

   (d) If $h(t) = t^5$ then $\lim_{t\to 2}\frac{h(t)-h(2)}{t-2} = 80$.

   (e) $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

(f) Let $f(x) = |x|$. Then, $f'(2)$ does not exist.

4. Find $f'(x)$ if it is known that

$$\frac{d}{dx}\left(f(2x)\right) = x^2.$$

5. Find $f'$ in terms of $g'$:

   (a) $f(x) = x^2 g(x)$,

   (b) $f(x) = g(x^2)$,

   (c) $f(x) = g(g(x))$.