Middlebury College

Calculus II: Fall 2017

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Practice Examination II

Trigonometric Identities and Integral Formulae
You may use the following identities and formulae without proof.

a.

$$\cos^2(x) + \sin^2(x) = 1$$
$$1 + \tan^2(x) = \sec^2(x)$$

b.

$$\begin{aligned} \sin(2x) &= 2\sin(x)\cos(x) \\ \cos(2x) &= \cos^2(x) - \sin^2(x) \\ &= 1 - 2\sin^2(x) \\ &= 2\cos^2(x) - 1 \end{aligned}$$

c.

$$\int \sec(x)dx = \log(\sec(x) + \tan(x)) + C$$
$$\int \tan(x)dx = \log|\sec(x)| + C$$

d.

$$\frac{d}{dx}(\tan(x)) = \sec^2(x)$$
$$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$$

e.

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \frac{1}{a}\arcsin\left(\frac{x}{a}\right) + C$$
$$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$

Instructions:

- Write your name on this exam and any extra sheets you hand in.
- Sign the Honor Code Pledge below.
- You will have 60 minutes to complete this Examination.
- You **must** attempt Problem 1.
- You **must** attempt **at least three** of Problems 2, 3, 4, 5.
- If you attempt all five problems then your final score will be the sum of your score for Problem 1 and the highest possible score obtained from three of the four remaining problems.
- There are 3 blank pages attached for scratchwork.
- **Calculators are not permitted.**
- **Explain your answers *clearly* and *neatly* and in *complete English sentences*.**
- State all Theorems you have used from class. To receive full credit you will need to justify each of your calculations and deductions coherently and fully.
- Correct answers without appropriate justification will be treated with great skepticism.

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1. (10 points) True/False:

   (a) Let $A = [-5, -1]$. Then, the function $f(x) = 3-(x+4)^2$ with domain A has an inverse function.

   (b) Using the inverse trigonometric substitution $2x = \sin(t)$, you determine

   $$\int f(x)dx = \sec(t) + \cos(2t) + t + C,$$

   for some function $f(x)$. Then,

   $$\frac{d}{dx}\left(\frac{1}{\sqrt{1-4x^2}} - 4x^2 + \arcsin(2x)\right) = f(x)$$

   (c) Let $a = \exp(b)$. Then,

   $$\int_1^b \frac{1}{t}dt = a.$$

   (d) The arc length of the curve $y = \frac{2}{3}x^{3/2}$ between $x = 1$ and $x = 3$ is $\frac{8}{3}$.

   (e)

   $$\int \frac{x}{\sqrt{(1+x^2)^5}}dx = -\frac{1}{3}\frac{1}{(1+x^2)^{3/2}} + C$$

2. Solve the following antiderivative problems. Show all your working and provide complete justification.

   (a) (10 points)

   $$\int \frac{1}{\sqrt{x^2+4x+5}}dx$$

   (b) (10 points)

   $$\int x\arctan(x)dx$$

3. Show all your working and provide complete justification. Let

   $$I_n = \int (\log(x))^n dx$$

   (a) (10 points) Let n be a natural number. Use integration by parts to show

   $$I_n = x(\log(x))^n - nI_{n-1}$$

   (b) (10 points) Let $e = \exp(1)$ be Euler's number. Using induction show that, for n a natural number,

   $$\int_1^e (\log(x))^n dx = ke + (-1)^{n-1}n!$$

   where k is an integer.

4. Show all your working and provide complete justification.

   (a) (10 points) Determine the partial fraction decomposition of the rational function

   $$f(x) = \frac{x^2+4x+1}{x^2(x^2+1)}$$

   (b) (10 points) Determine the integral

   $$\int f(x)dx$$

5. (20 points) Find the area of the surface of revolution (about the x-axis) obtained from the graph of the function $f(x) = (x-1)^3$ between $x = 1$ and $x = 2$.