Middlebury College

Calculus II: Fall 2017

Contact: `gmelvin@middlebury.edu`

Practice Examination I

Instructions:

- You *must* attempt Problem 1.
- Please attempt at least three of Problems 2,3,4,5.
- If you attempt all five problems then your final score will be the sum of your score for Problem 1 and the scores for the three remaining problems receiving the highest number points.
- Calculators are not permitted.

---

1. (10 points) True/False:

   (a) Let (a_n) be a sequence. If the sequence of even terms $(a_2, a_4, a_6, \ldots)$ is convergent with limit L then (a_n) is convergent with limit L.

   (b) Let $\sum a_n$ be a series. If the associated equence of partial sums (s_m) is decreasing and bounded then the sequence (a_n) is convergent.

   (c) Let (a_n) be a bounded sequence. Suppose that there exists N such that the sequence $(a_n)_{n\geq N}$ is decreasing. Then, (a_n) is convergent.

   (d) Let $\sum a_n$ be a series such that $a_n > 0$. Let (s_m) be the associated sequence of partial sums. If there exists K such that $s_m < K$, for $m = 1, 2, 3, \ldots$, then $\sum a_n$ is convergent.

   (e) Let $\sum(-1)^n b_n$, where $b_n > 0$, be an alternating series. If $\sum b_n$ is convergent then $\sum(-1)^n b_n$ is convergent.

2. Determine if the following sequences converge or diverge. If the sequence converges determine the limit. Give a careful explanation of your solution.

   (a) (10 points)
   $$\left(\frac{\sin(\frac{1}{n})}{2^n}\right)_{n\geq 1}$$

   (b) (10 points)
   $$\left(\frac{n}{2 + (-1)^n}\right)_{n\geq 1}$$

3. (20 points) Consider the sequence (a_n), where
   $$a_n = \frac{2^n}{n!}, \qquad n = 1, 2, 3, \ldots$$

   (a) Show that (a_n) is a decreasing sequence.

   (b) Determine an upper and lower bound for the sequence (a_n).

   (c) Explain carefully why the series (a_n) is convergent.

(d) Determine $\lim_{n\to\infty} a_n$.

4. (20 points) Determine if the following series is convergent or divergent. If convergent you do not need to determine its limit. Justify your answer carefully.

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{7^n - 2^n}$$

5. Determine whether the following series is absolutely convergent, conditionally convergent or divergent. If convergent you do not need to determine its limit. Justify your answer carefully.

$$\sum_{n=1}^{\infty} (-1)^{n+1} n\pi^{-n^2} = \pi^{-1} - 2\pi^{-4} + 3\pi^{-9} - \dots$$