CALC II SEQUENCES... & SERIES

EXAM #1 REVEIW

RIEEMAN SUM ⇒ $S_n = \frac{(b-a)}{n}(f(x_1) + f(x_2) + f(x_3) + \dots + f(x_{n-1}) + f(x_n))$

first term, last term, function, n= # of sub sections

NATURAL NUMBERS => $\mathbb{N}$ => all positive integers (whole numbers), counting #s 1, 2, 3 ...

Given a property P, a function will satisfy 1 of 3 conditions:

(I) property P is true (holds) for ALL but FINITELY many n
↳ it is true for INFINITELY many

(II) property P is false (it doesn't hold) for ALL except FINITELY MANY
↳ it is false of INFINITELY many

(III) neither I or II

→ IF I or II hold, there is a number N where $n \geq N$ for which ALL the following #s are true/false for property P

$\boxed{\text{SEQUENCE}}$ - a collection of outputs, an infinitely long list

$$a_n = (a_1, a_2, a_3 \dots a_n \dots)$$

↳ a SEQUENCE is CONVERGENT with a LIMIT L if, for ~~a given~~ any $\epsilon > 0$ (basically any small number), the property $P_{\epsilon, L}$ holds for the sequence a_n a $n \to \infty$

↳ the property is that the value of a_n falls with ϵ of the limit L

=> IF a_n is a sequence convergent with limit L, for any $\epsilon > 0$ there is a natural number N such that $n \geq N \Rightarrow |a_n - L| < \epsilon$

Calc II SEQUENCES... & SERIES THINGS TO KNOW

LIMIT LAWS for Sequences

Let a_n and B_n be convergent sequences & C is a constant

$$\lim_{n\to\infty}(a_n \pm B_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} B_n$$

$$\lim_{n\to\infty} c\,a_n = C \lim_{n\to\infty} a_n$$

$$\lim_{n\to\infty} a_n B_n = \left(\lim_{n\to\infty} a_n\right)\left(\lim_{n\to\infty} B_n\right)$$

$$\lim_{n\to\infty} \frac{a_n}{B_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} B_n}$$

$$\lim_{n\to\infty} a_n^r = \left(\lim_{n\to\infty} a_n\right)^r$$

ROOT RULES:

$\lim_{n\to\infty} \sqrt[n]{C} = 1$ for any constant $C>0$

$\lim_{n\to\infty} \sqrt[n]{n^p} = 1$ for any $p>0$

$\lim_{n\to\infty} \sqrt[n]{f(n)} =$ for any non zero polynomial function

$\lim_{n\to\infty} \sqrt[n]{n!} = +\infty$

Let $r>0$. Then $\lim_{n\to\infty} \frac{1}{n^r} = 0$

IF a_n is CONVERGENT then a_n is BOUNDED

IF a_n is BOUNDED then it is CONVERGENT as long as it is also MONOTONIC

A ~~S~~SERIES is the LIMIT of a SEQUENCES OF PARTIAL SUMS

$$\sum_{n=1}^{\infty} a_n = \lim_{m\to\infty} S_m$$

where a_n is a SEQUENCE

& S_m is the partial sum of a_n

$S_m = a_1 + a_2 + a_3 + \ldots + a_m$

IF THE SERIES $\sum_{n=1}^{\infty}$ CONVERGES (has a limit, value)

THEN $\lim_{n\to\infty} a_n = 0$

↳ HOWEVER! IF this is true, convergence is not implicit

↳ even if a_n has a FINITE limit, unless it is $L=0$, ~~&~~ $\sum a_n$ will DIVERGE!

THE SQUEEZE THEOREM

Let a_n, B_n, C_n be sequences where B_n and C_n are CONVERGENT with limit L

$$\lim_{n\to\infty} B_n = L = \lim_{n\to\infty} C_n$$

Then, if there is a number $n_0 \geq 1$ where, if $n \geq 1$

$$B_n \leq a_n \leq C_n$$

Then the sequence a_n MUST BE convergent with the limit L

$$\lim_{n\to\infty} a_n = L$$

THE MONOTONIC + BOUDED THEOREM

IF a_n is ~~x~~ **MONOTONIC** (strictly increasing or decreasing)

AND

BOUNDED ($\underset{\text{little}}{n} \leq a_n \leq \underset{\text{BIG}}{N}$)

Then a_n is CONVERGENT

THE GEOMETRIC* SERIES THEOREM

let $-1 < r < 1$ then the geometric series $\sum_{n=1}^{\infty} r^n$ is CONVERGENT with limit $L = \frac{r}{1-r}$

ex:

$$\sum_{n=1}^{\infty} \frac{3}{10^n} = 3 \sum_{n=1}^{\infty} \frac{1}{10^n} = 3 \sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^n$$

$r = \frac{1}{10}$

* A series $\sum_{n=1}^{\infty} a_n$ is a geometric series IF there is a real number r so that for every $n = 1, 2, 3 \ldots$

$$\frac{a_{n+1}}{a_n} = r$$

THE P SERIES TEST

Given a series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ (where p is a real #)

Then

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ **CONVERGES IF** $p \geq 2$ or $p > 1$

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ **DIVERGES IF** $p \leq 1$

THE DCT: DIRECT COMPARISON TEST

Let $\sum a_n$ and $\sum B_n$ be series having positive terms.

IF $\sum B_n$ is CONVERGE and $a_n \leq B_n$ **THEN** $\sum a_n$ is also CONVERGENT

IF $\sum B_n$ is DIVERGENT and $a_n \geq B_n$ **THEN** $\sum a_n$ is also DIVERGENT

SEQUENCES

SERIES (To show that a series is convergent one of the conditions is that the sequence is

LIMIT COMPARISON TEST

Suppose that $\sum a_n$ and $\sum B_n$ are series with positive terms.

If the **SEQUENCE** $\left(\frac{a_n}{B_n}\right)$ is convergent

and $\lim_{n\to\infty}\left(\frac{a_n}{B_n}\right) = c > 0$ ← a positive number

then BOTH **series** either CONVERGE or diverge!

ALTERNATING SERIES TEST

Let $\sum (-1)^n \cdot B_n$ be an alternating series where $B_n > 0$ for $n = 1, 2, 3, \ldots$

IF B_n is DECREASING

AND $\lim_{n\to\infty} B_n = 0$

THEN the series $\sum (-1)^n \cdot B_n$ is CONVERGENT

ALTERNATING SERIES ESTIMATION THEOREM:

If the above is true

then $\sum (-1)^n B_n = L$

then $|L - s_n| \leq B_n + 1$

ABSOLUTE CONVERGENCE

IF $\sum |a_n|$ is convergent,

THEN $\sum a_n$ MUST also be convergent

RATIO TEST

Let $\sum a_n$ be a series where $a_n \neq 0$

IF $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$

THEN the series $\sum a_n$ is ABSOLUTELY CONVERGENT

IF $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$ OR $+\infty$

THEN $\sum a_n$ is DIVERGENT

IF $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L = 1$

THEN the test is INCONCLUSIVE

ROOT TEST

Let $\sum a_n$ be a series where $a_n \neq 0$

IF $\lim_{n\to\infty}\sqrt[n]{|a_n|} = L < 1$

THEN the series $\sum a_n$ is ABSOLUTELY CONVERGENT

IF $\lim_{n\to\infty}\sqrt[n]{|a_n|} = L > 1$ or $+\infty$

THEN the series $\sum a_n$ is DIVERGENT

IF $\lim_{n\to\infty}\sqrt[n]{|a_n|} = L = 1$

THEN the test is INCONCLUSIVE