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Calculus I Review

It will be assumed that you have some exposure to and practice with the following topics.

1. The notion of the *limit* $\lim_{x \to a} f(x)$ of a real-valued function $f(x)$ as x tends towards a, and the definition of a *continuous function* in terms of limits.

2. The following Theorems: let $f(x)$, $a \leq x \leq b$, be a continuous function.

   (a) (MIN/MAX THEOREM) There exists real numbers m and M such that $m \leq f(x) \leq M$. Moreover, there exists $a \leq y \leq b$ and $a \leq z \leq b$ such that $f(y) = m$ and $f(z) = M$.

   (b) (INTERMEDIATE VALUE THEOREM) Suppose that $f(a) = c$, $f(b) = d$ so that $c < d$ and let $c < e < d$. Then, there exists u with $a < u < b$ such that $f(u) = e$.

3. The definition of the *derivative* of $f(x)$ at $x = a$ in terms of limits

$$f'(a) \overset{def}{=} \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

   The notation f', $\frac{d}{dx}f(x)$, $\frac{d}{dx}f$, $\frac{df}{dx}$, and the interpretation of the derivative as the *rate of change* of $f(x)$ near to $x = a$.

4. The following theorems on differentiation: let $f(x), g(x)$ be functions, C a constant.

   (a) (LINEARITY) $(f+g)'(x) = f'(x) + g'(x), \quad (Cf)'(x) = Cf'(x)$

   (b) (PRODUCT RULE) $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$

   (c) (QUOTIENT RULE) $\left(\frac{f}{g}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{f(x)^2}$

   (d) (CHAIN RULE) $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

5. The following elementary derivatives:

   (a) Let $n \neq 0$ be an integer. Then, $\frac{d}{dx}(x^n) = x^{n-1}$

   (b) $\frac{d}{dx}\cos(x) = -\sin(x), \quad \frac{d}{dx}\sin(x) = \cos(x)$

6. (ANTIDERIVATIVE PROBLEM) Given $f(x)$, does there exists an *antiderivative* $F(x)$ such that $F'(x) = f(x)$? An antiderivative $F(x)$ of $f(x)$ is frequently called an *indefinite integral (of $f(x)$)*, denoted $\int f(x)dx$, or simply $\int f$.

7. Determining antiderivatives of certain elementary functions.

8. The definition of the *definite, or Riemann, integral* $\int_a^b f(x)dx$ using Riemann sums.

9. (FUNDAMENTAL THEOREM OF CALCULUS) Let $f(x)$, $a \leq x \leq b$, be continuous.

   (a) The function

$$F(x) = \int_a^x f(u)du, \quad a \leq x \leq b,$$

   is an antiderivative of $f(x)$.

   (b) The definite integral $\int_a^b f(x)dx$ can be evaluated using an antiderivative $F(x)$ of $f(x)$: namely,

$$\int_a^b f(x)dx = F(b) - F(a).$$