Math 121C1/E: Single-Variable Calculus
Fall 2018

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Supplementary References:
- *Calculus*, Hughes-Hallet et al, Section 2.3

Keywords: *the derivative function, power law, linearity, Leibniz notation*

The Derivative Function contd.

Some Derivative Rules

- **Power Law:** if $f(x) = x^n$, where $n \geq 0$ is an integer, then $f'(x) = nx^{n-1}$
- **Linearity:** Let $f(x), g(x)$ be functions with derivatives $f'(x), g'(x)$.
  - $(f \pm g)'(x) = f'(x) \pm g'(x)$,
  - $(cf)'(x) = cf'(x)$, for any constant c.

Notation: The '*prime notation*' $f'(x)$ is due to Lagrange (1736-1813). We also write

$$\frac{d}{dx}f(x) = f'(x)$$

If $y = f(x)$ then write

$$\frac{dy}{dx} = \frac{d}{dx}f(x) = f'(x)$$

This is called **Leibniz notation**. For example, the above rules become

- **Power Law:** if $f(x) = x^n$, where $n \geq 0$ is an integer, then $\frac{d}{dx}(x^n) = nx^{n-1}$
- **Linearity:** Let $f(x), g(x)$ be functions with derivatives $f'(x), g'(x)$.
  - $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$,
  - $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$, for any constant c.

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1. $f(x) = 7x^3 - 2x^2 + 9$, $\frac{d}{dx}f(x) = 21x^2 - 4x$.
2. $y = x^{32} - x^{10} + 2x^8 + \pi^2 x^3 - 3$, $\frac{dy}{dx} = 32x^{31} - 10x^9 + 16x^7 + 3\pi^2 x^2$.

Remark: We can now compute derivatives of *any* polynomial function.

Example: Let $f(x) = \sqrt{x} = x^{1/2}$. What's the derivative?

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Hence,

$$\frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{-1/2}$$

This provides some evidence towards the following:

Fact: Let r be any real number. Then,

$$\frac{d}{dx}x^r = rx^{r-1}$$

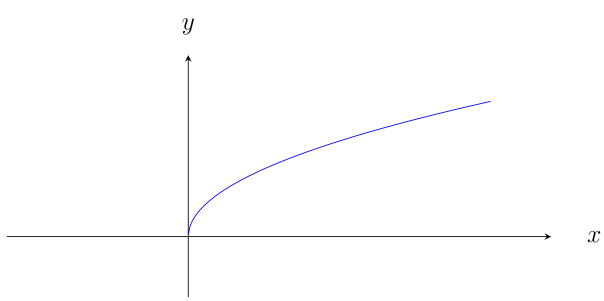
Example:

1. Let $y = \frac{3}{x^2} + 2\sqrt[3]{x^3} - 3 = 3x^{-2} + 2x^{3/2} - 3$. Then,

$$\frac{dy}{dx} = -6x^{-3} + 3x^{1/2} = -\frac{6}{x^3} + 3\sqrt{x}$$

2. Let $f(x) = \frac{2x^3 - 5\sqrt{x}}{x^2} = 2x - 5x^{-3/2}$. Then, $f'(x) = 2 + \frac{15}{2}x^{-5/2} = 2 + \frac{15}{2\sqrt{x^5}}$.

Observe: If $f(x) = \sqrt{x}$ then $f'(x) = \frac{1}{2\sqrt{x}}$ is undefined at $x = 0$.



This corresponds to fact that the tangent line to the graph $y = \sqrt{x}$ at $x = 0$ is vertical i.e. its slope is undefined.