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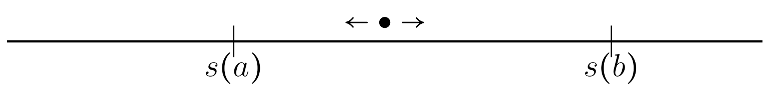
September 14 Summary

Supplementary References:
- *Calculus*, Hughes-Hallet et al, Section 1.8

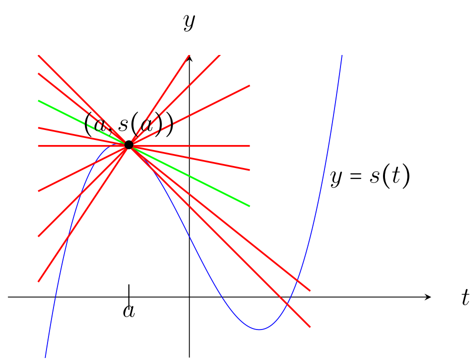
Keywords: *limits*

Limits

• **Recall:** Let $s(t)$ denote the position of an object moving on a straight line at time $t$secs.



The **instantaneous velocity** of the object at time $t = a$ equals the slope of the **tangent line** to $y = s(t)$ at $(a, s(a))$



How to compute this slope? Use secant lines through $(a, s(a))$ and $(b, s(b))$: compute the slopes of these secant lines as b gets closer and closer, but not equal, to a.

Limits:
• Let $f(x)$ be a function. Write $\lim_{x \to a} f(x) = L$ if the *values of* $f(x)$ *approach* L *as* x *gets closer and closer,* **but not equal**, *to* a. Say: *"limit of* f *as* x *approaches* a *is* L*"*
• Also write: $f(x) \to L$ as $x \to a$.

Example:

1. $f(x) = 2x$:

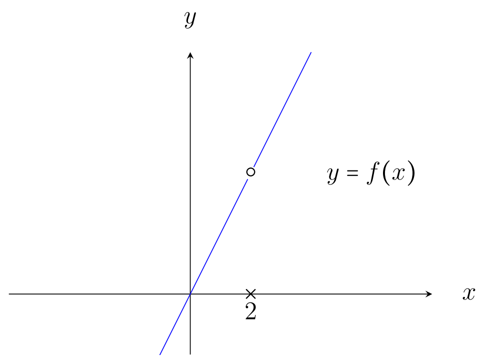
x	$f(x)$
1.9	3.8
1.99	3.98
1.999	3.9998
2.1	4.2
2.01	4.02
2.001	4.002

As x gets close to $a = 2$, $f(x)$ is getting closer to 4. Write $\lim_{x \to 2} 2x = 4$.

- **Remark:** nowhere in the above example have we determined $f(2)$.

2. Define

$$f(x) = \begin{cases} 2x, & x \neq 2 \\ 0, & x = 2 \end{cases}$$



To determine $\lim_{x \to 2} f(x)$ we proceed exactly as above: we determine $\lim_{x \to 2} f(x) = 4 \neq f(2)$!