Math 121C1/E: Single-Variable Calculus
Fall 2018

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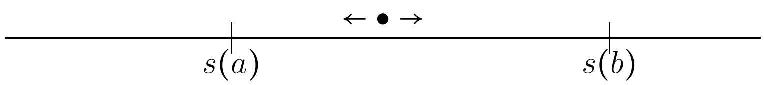
Supplementary References:
- *Calculus*, Hughes-Hallet et al, Sections 2.1

Keywords: *average velocity, instantaneous velocity, tangent line*

Instantaneous Velocity

Consider motion of an object along a straight line:

$$t = \text{ time in secs}, \quad s(t) = \text{ position at time } t$$



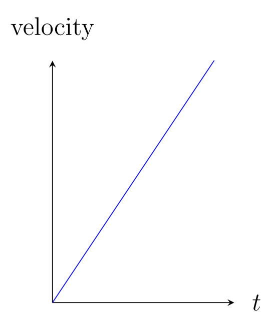
• the **average velocity** over the interval $a \leq t \leq b$ is

$$\frac{\text{change in position}}{\text{change in time}} = \frac{s(b) - s(a)}{b - a}$$

• **Example:** Drop a bowling ball from the Tower of Pisa. Galileo's Law states that, after t seconds, the ball has fallen $s(t) = 4.9t^2$ metres.

$$\text{ave. velocity over } 0 \leq t \leq 5 = \frac{s(5) - s(0)}{5 - 0} = \frac{4.9(5^2) - 0}{5} = 24.5ms^{-1}$$

• Objects falling under influence of gravity (neglecting air resistance) are accelerating at a constant rate i.e. the velocity is increasing at a fixed rate



• Ave. velocity on interval $0 \leq t \leq 5$ does not represent true velocity at $t = 5$secs. To approximate true velocity at $t = 5$secs consider a shorter time interval:

$$\text{ave. velocity over } 5 \leq t \leq 5.1 = \frac{s(5.1) - s(5)}{5.1 - 5} = \frac{4.9(5.1)^2 - 4.9(5)^2}{0.1} = 49.49ms^{-1}$$

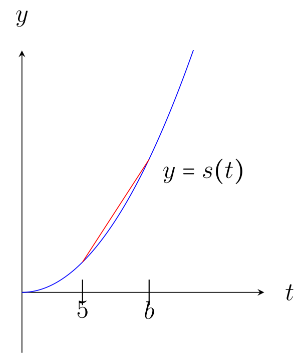
Similar calculations over shorter time intervals give:

Interval	Ave. velocity (ms^{-1})
$5 \le t \le 5.1$	49.49
$5 \le t \le 5.01$	49.049
$5 \le t \le 5.001$	49.0049

- Can reinterpret average velocity:

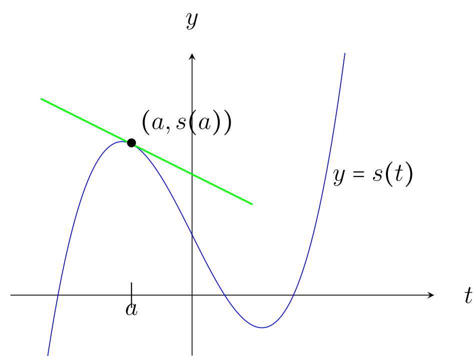
$$\text{ave. velocity over } 5 \le t \le b = \frac{s(b) - s(5)}{b - 5}$$

This gives slope of **secant line** through $(5, s(5))$ and $(b, s(b))$ on graph of $s(t) = 4.9t^2$



- **Goal:** Understand true velocity at $t = 5$secs.
- **PRoblem:** can't input $b = 5$ into average velocity formula i.e. would divide by 0
- **Strategy:** consider average velocities as b gets closer and closer to 5.

- Let $s(t)$ = position at time t of object moving in a straight line. Define **instantaneous velocity** of the object at $t = a$ to be the slope of the **tangent line** of the graph $y = s(t)$ at $(a, s(a))$



- Find the slope of tangent line L using secant lines through $(a, s(a))$ i.e. for any b consider the slope of the secant line passing through $(a, s(a))$ and $(b, s(b))$. Now let b get closer and closer to a, without allowing b to ever equal a.