Math 121C1/E: Single-Variable Calculus
Fall 2018

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September 11 Summary

Supplementary References:
- *Calculus*, Hughes-Hallet et al, Sections 1.2, 1.5

Keywords: *exponential functions, logarithm functions, trigonometric functions*

Exponential & Logarithm Functions, Trigonometric Functions

- An **exponential function with base** $a > 0$ is a function $P(x) = Ka^x$, K constant.
- A very important example occurs when $a = e = 2.71828\ldots$ (**Euler's number**)

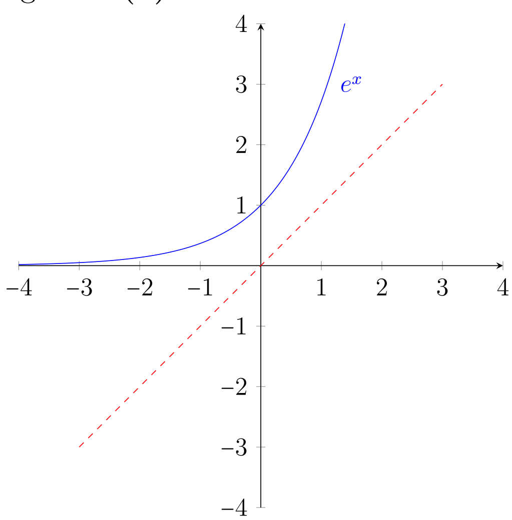
- The graph of $P(x) = e^x$ passes HLT $\implies$ $P(x)$ has an inverse. We call this inverse the *natural logarithm*; denoted $\ln(x)$.
- **Properties:**

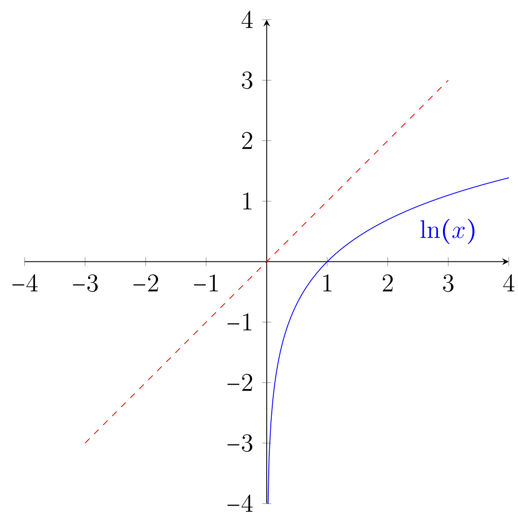
$$e^{\ln(x)} = x, \quad \ln(e^x) = x$$

and

$$\ln(x) = c \quad \Leftrightarrow \quad x = e^c$$

- domain of $\ln(x)$ = range of e^x = positive real numbers.
- range of $\ln(x)$ = domain of e^x = $\mathbb{R}$.





- **Fact:** for any $a > 0$, there's constant r satisfying $a^x = e^{rx}$

Why? $a = e^{\ln(a)} \implies a^x = \left(e^{\ln(a)}\right)^x = e^{\ln(a)x}$ i.e. take $r = \ln(a)$.

- **Some log rules:** Let

$$\ln(A) = c \quad \Leftrightarrow A = e^c$$

$$\ln(B) = d \quad \Leftrightarrow B = e^d$$

Set

$$\ln(AB) = z \quad \Leftrightarrow \quad AB = e^z \quad \Leftrightarrow \quad e^c e^d = e^z \quad \Leftrightarrow \quad e^{c+d} = e^z$$

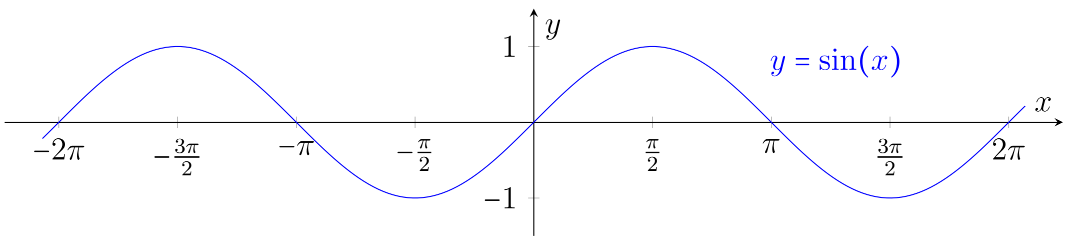
Apply ln to last equation:

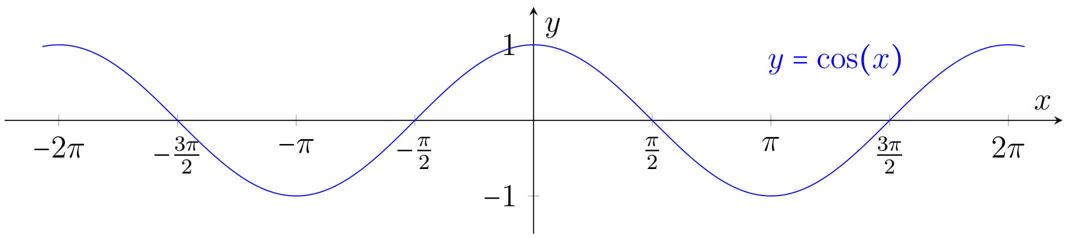
$$c + d = \ln(e^{c+d}) = \ln(e^z) = z \quad \Longrightarrow \quad \ln(A) + \ln(B) = \ln(AB)$$

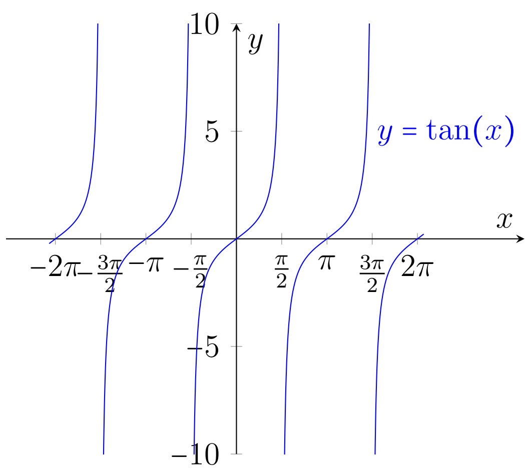
- Can also show

$$\ln(A^r) = r\ln(A)$$
$$\ln\left(\frac{A}{B}\right) = \ln(A) - \ln(B)$$

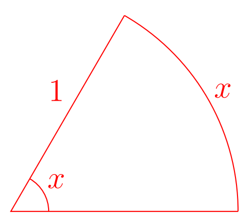
Trigonometric functions:







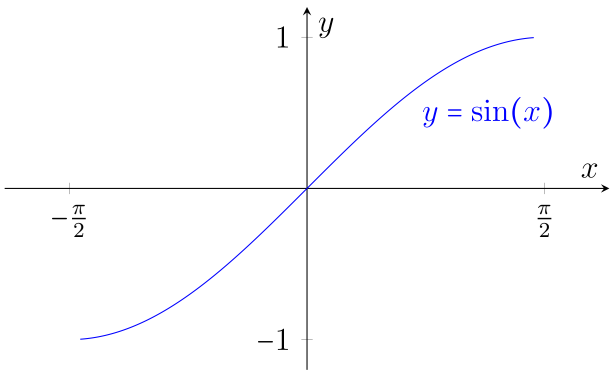
- x measured in radians:

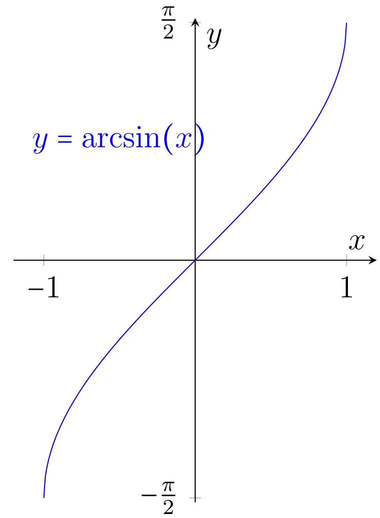


The length of the arc of the unit circle is equal to the angle (measured in radians) that it subtends with the horizontal.

Inverse trigonometric functions

Graph of $f(x) = \sin(x)$ does not pass HLT. However, if we restrict domain to $-\pi/2 \leq x \leq \pi/2$ then it does:





Call inverse function $\arcsin(x)$:

$$\sin(\arcsin(x)) = x, \quad \arcsin(\sin(x)) = x$$

$$\arcsin(x) = y \quad \Leftrightarrow \quad x = \sin(y)$$

In words, $\arcsin(x)$ *is the angle (in radians) whose sine is* x i.e. *the arc whose sine is* x