Colby

Math 121C1/E: Fall 2018
Midterm 2 Review Problems
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Some Review Problems

Compute the local linearisation of $f(x)$ at $x = a$

1. $f(x) = \sin(x)$, $x = \pi$.
2. $f(x) = e^{-x^2}$, $x = \frac{1}{\sqrt{2}}$.

Solution:

1. $L(x) = -x + \pi$
2. $L(x) = \frac{1}{\sqrt{e}} - \frac{\sqrt{2}}{\sqrt{e}}\left(x - \frac{1}{\sqrt{2}}\right)$

Determine the derivative.

1. $f(x) = e^{e^x}$
2. $y = \cos(e^x + x^2)$
3. $g(t) = (\sin(t) + 2t^3)^{10}$

Solution:

1. $f'(x) = e^x e^{e^x} = e^{x+e^x}$
2. $\frac{dy}{dx} = -\sin(e^x + x^2)(e^x + 2x)$
3. $g'(t) = 10(\sin(t) + 2t^3)^9(\cos(t) + 6t^2)$

Using logarithmic differentiation, determine the derivative

1. $y = (\cos(x) + 2)^x$
2. $y = \left(\frac{e^x+1}{x^2+2}\right)^{5/3}$
3. $y = x^{\sqrt{x}}$

Solution:

1. $\frac{dy}{dx} = (\cos(x) + 2)^x \left(\ln(\cos(x) + 2) - \frac{x\sin(x)}{\cos(x)+2}\right)$
2. $\frac{dy}{dx} = \frac{5}{3}\left(\frac{e^x+1}{x^2+2}\right)^{5/3}\left(\frac{e^x}{e^x+1} - \frac{2x}{x^2+2}\right)$
3. $\frac{dy}{dx} = x^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\ln(x) + \frac{1}{\sqrt{x}}\right)$

Use L'Hopital's Rule to determine the limit

1. $\lim\limits_{x\to 1} \dfrac{x^{1/3} - 1}{x^{2/3} - 1}$

2. $\lim_{x \to 0^+} \frac{1}{x} - \frac{1}{\sin(x)}$

3. $\lim_{x \to \infty} (1 + \sin(3/x))^x$ (*Hint: let* $y = (1 + \sin(3/x))^x$ *and apply* ln)

Solution:

1. 1/2

2. 0

3. e^3

Miscellaneous

1. How fast is the surface area of a cube changing when the volume of the cube is 64cm^3 and is increasing at 2cm^3 per second?

2. A box is to be made from a rectangular sheet of cardboard 70cm by 150cm by cutting equal squares out of the four corners and bending up the resulting four flaps to make the sides of the box (the box has no top). What is the largest possible volume of the box?

Solution:

1. 2cm^3 per second

2. Numbers were too big.... You should have $V = x(70 - 2x)(150 - 2x)$; solve $\frac{dV}{dx} = 0$