Contact: `gwmelvin@colby.edu`

Practice Midterm 2

Disclaimer: This Practice Midterm consists of problems of a similar difficulty as will be on the actual midterm. However, **problems on the actual midterm may or may not be quite different in nature, and may or may not focus on different course material**. However, the actual midterm will have a similar format: one true/false problem, one short-answer problem, three long-answer problems.

---

1. True/False (no justification required)

   (a) $\lim_{x \to 0^+} \dfrac{\ln(x^2)}{x} = 1$

   (b) Let $f(x) = e^x + 5$, where $-1 \leq x \leq 10$. Then, $x = 10$ is a local maximum of $f(x)$.

   (c) If $f''(p) = 0$ then p is an inflection point.

   (d) The linear approximation of $f(x) = \cos(x)$ at $x = \pi/2$ is 0.

   (e) Let $y = \sin(\cos(x))$. Then, $\frac{dy}{dx} = \sin(\cos(x)) + \cos(\sin(x))$.

2. (a) Let $g(t) = e^{\cos(t^3)}$. Determine $g'(t)$.

   (b) Let $y = (1 - 2x)^{e^x}$. Using logarithmic differentiation, determine $\frac{dy}{dx}$.

3. A piece of wire having length 10m is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle.

   (a) Let x be the side length of the square. Show that the total area A of the square and triangle can be expressed as
   $$A = x^2 + \frac{\sqrt{3}}{4}\left(\frac{10 - 4x}{3}\right)^2$$
   Hint: the area of an equilateral triangle having side length a is $\frac{\sqrt{3}}{4}a^2$.

   (b) Determine the side length of the square giving the largest possible area A.

4. At 1pm ship A is 25km due north of ship B. If Ship A is sailing west at a rate of 16km/h and ship B is sailing south at 20km/h, find the rate at which the distance between the two ships is changing at 1.30pm.

5. Let $f(x) = xe^{-x^2}$.

   (a) Determine the local maxima/minima of $f(x)$.

   (b) Determine the inflection points of $f(x)$.

   (c) Using L'Hopital's Rule, show that $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to -\infty} f(x) = 0$.

   (d) Using what you've found, sketch the graph of $f(x)$.