121 REVIEW PROBLEMS SOLUTIONS

1.) Typo: $f(x) = x^2 \sin(x)$

$$f'(x) = 2x\sin(x) + x^2 \sin(x)$$

2.) $f'(x) = \dfrac{-1}{2\sqrt{x^3}} + \dfrac{3}{5} \cdot \dfrac{1}{\sqrt[5]{x^8}}$

3) $f'(x) = \dfrac{1+\cos^3(x)}{\cos(x)(1+\cos(x))^2}$

4) $f'(x) = \dfrac{8x^3}{(x^4+1)^2}$

5) $f'(x) = \dfrac{-1}{(x-1)^2}$

6) $f'(x) = 2\cos(x)\sin(x) + 2x\left(\cos^2(x) - \sin^2(x)\right)$.

1.) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{5(x+h)^2 + 5(x+h) - x^2 - 5x}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 + \cancel{5x} + 5h - \cancel{5x^2} - \cancel{5x}}{h}$$

$$= \lim_{h\to 0} 10x + 5 + 5h = 10x + 5$$

2.) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\sqrt{3-5(x+h)} - \sqrt{3-5x}}{h} \cdot \frac{\sqrt{3-5(x+h)} + \sqrt{3-5x}}{\sqrt{3-5(x+h)} + \sqrt{3-5x}}$$

$$= \lim_{h \to 0} \frac{3-5(x+h) - (3-5x)}{h\left(\sqrt{3-5(x+h)} + \sqrt{3-5x}\right)}$$

$$= \lim_{h \to 0} \frac{-5}{\sqrt{3-5(x+h)} + \sqrt{3-5x}}$$

$$= \frac{-5}{2\sqrt{3-5x}}$$

1.) $f(x) = x^6$, $a = 2$

2) $f(x) = x^{3/2}$, $a = 1$

1.) Note $\frac{x^2 - 9}{x^2 + 2x - 3} = \frac{(x-3)(x+3)}{(x-1)(x+3)}$

$$\lim_{x \to 3} \frac{x^2 - 9}{x^2+2x-3} = \lim_{x \to 3} \frac{x-3}{x-1}$$

let $f(x) = \frac{x-3}{x-1}$. Then, $f(x)$ conts on it domain.

$x \neq 1$

Since $x = 3$ is in domain, then continuity at $x = 3$ gives

$$\lim \frac{x^2 - 9}{x^2+2x-3} = f(3) = \frac{0}{2} = 0.$$

2.) The function $f(x) = \dfrac{x^2+9}{x^2+2x+3}$ is conts

on its domain. Since $x = 1$ is in domain

Then continuity at $x=1$ gives

$$\lim_{x \to 1} \frac{x^2+9}{x^2+2x+3} = f(1) = \frac{10}{6} = \frac{5}{3}$$

3) <u>Note:</u>

$$|x-4| = \begin{cases} x-4 & , \ x \geq 4 \\ -x+4 & , \ x < 4. \end{cases}$$

$$\lim_{x \to 4^+} \frac{|x-4|}{x-4} = \lim_{x \to 4^+} \frac{x-4}{x-4} = \lim_{x \to 4^+} 1 = 1.$$

by LL5