

SOME REVIEW PROBLEMS

Determine the derivative $f'(x)$.

1.

$$f(x) = x^2 \sin(x)$$

2.

$$f(x) = \frac{1}{\sqrt{x}} - \frac{1}{\sqrt[5]{x^3}}$$

3.

$$f(x) = \frac{\tan(x)}{1 + \cos(x)}$$

4.

$$f(x) = \frac{x^4 - 1}{x^4 + 1}$$

5.

$$f(x) = \frac{1}{1 - x^{-1}}$$

6.

$$f(x) = 2x \cos(x) \sin(x)$$

Determine $f'(x)$ using the limit definition.

1.

$$f(x) = x^2 + 5x$$

2.

$$f(x) = \sqrt{3 - 5x}$$

Find a function $f(x)$ and a real number a so that

1.

$$\lim_{h \rightarrow 0} \frac{(2+h)^6 - 64}{h} = f'(a)$$

2.

$$\lim_{h \rightarrow 0} \frac{\sqrt{(1+h)^3} - 1}{h} = f'(a)$$

Compute the limit using Limit Laws or otherwise. You may need to perform some algebra to simplify the expression

1.

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 + 2x - 3}$$

2.

$$\lim_{x \rightarrow 1} \frac{x^2 + 9}{x^2 + 2x + 3}$$

3.

$$\lim_{x \rightarrow 4^+} \frac{|x - 4|}{x - 4}$$

Limit Laws: Let $f(x)$, $g(x)$ be functions. Assume that $\lim_{x \rightarrow c} f(x)$ and $\lim_{x \rightarrow c} g(x)$ both exist.

$$(\text{LL1}) \quad \lim_{x \rightarrow c} bf(x) = b \left(\lim_{x \rightarrow c} f(x) \right), \text{ for any constant } b.$$

$$(\text{LL2}) \quad \lim_{x \rightarrow c} f(x) \pm g(x) = \lim_{x \rightarrow c} f(x) \pm \lim_{x \rightarrow c} g(x)$$

$$(\text{LL3}) \quad \lim_{x \rightarrow c} f(x)g(x) = \left(\lim_{x \rightarrow c} f(x) \right) \left(\lim_{x \rightarrow c} g(x) \right)$$

$$(\text{LL4}) \quad \lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)}, \text{ provided } \lim_{x \rightarrow c} g(x) \neq 0.$$

$$(\text{LL5}) \quad \text{For any constant } k, \lim_{x \rightarrow c} k = k.$$

$$(\text{LL6}) \quad \lim_{x \rightarrow c} x = c.$$